

# Compressed Sensing Detector Design for Space Shift Keying in MIMO Systems

Chia-Mu Yu, Sung-Hsien Hsieh, Han-Wen Liang, Chun-Shien Lu, *Member, IEEE*,  
Wei-Ho Chung, *Member, IEEE*, Sy-Yen Kuo, *Fellow, IEEE* and Soo-Chang Pei, *Fellow, IEEE*

**Abstract**—Space shift keying (SSK) modulation and its extension, the generalized SSK (GSSK), present an attractive framework for the emerging large-scale MIMO systems in reducing hardware costs. In SSK, the maximum likelihood (ML) detector incurs considerable computational complexities. We propose a compressed sensing based detector, NCS, by formulating the SSK-type detection criterion as a convex optimization problem. The proposed NCS requires only  $O(n_t N_r N_t)$  complexity, outperforming the  $O(N_r N_t^{n_t})$  complexity in the ML detector, at the cost of slight fidelity degradation. Simulations are conducted to substantiate the analytical derivation and the detection accuracy.

**Index Terms**—Detector, space shift keying, compressed sensing, MIMO.

## I. INTRODUCTION

SPATIAL modulation (SM) [7], which encodes information in the combination of antenna indices and the conventional phase and amplitude, has attracted research attention in recent years. The SM facilitates energy efficiency and reduced hardware costs for multiple-input and multiple-output (MIMO) systems. As a simplified variation of SM, the space shift keying (SSK) modulation [5] activates only one antenna at any time instant and encodes information in antenna indices only. Although the usage of only antenna indices limits the information rate in each modulation symbol, such modulation has been shown to yield superior performance and could be an attractive option for the emerging large-scale MIMO systems due to the following advantages: 1) The number of required RF chains is reduced from the number  $N_t$  of transmit antennas to one, enabling the reduction of Inter-Channel Interference and hardware expenses. 2) The information is contained entirely in the indices of activated transmitting antennas, resulting in the dramatic reduction of detection complexity. 3) Only one antenna needs to be activated, which significantly reduces the

requirement of Inter-Antenna Synchronization. More details about SSK modulation can be found in [9], [10].

**Survey and Challenges:** A disadvantage of SSK is the relatively small-sized modulation alphabet and therefore the reduced attainable symbol rates compared to conventional modulations. Generalized SSK (GSSK) [6] is a variant of SSK, in which there are  $n_t$ , instead of only one in SSK, concurrently activated transmit antennas at a symbol instant. Hamming code-aided SSK (HSSK) [1] proposed a design with a varying number of activated transmit antennas. GSSK can effectively enhance the transmission rate while HSSK offers better transmission rate, performance, and power tradeoff. In spite of the relatively lightweight computational overhead for detection in SSK-type modulations, GSSK and HSSK require higher detection complexity for higher transmission rate, incurring the prohibitive detection complexity  $O(N_r N_t^{n_t})$  and  $O(\sum_{i=1}^{n_t} N_r N_t^i)$  respectively, where  $n_t^U$  denotes the maximum allowable number of activated antennas. The detection complexity tremendously increases particularly in the high-rate large MIMO systems with large  $N_t$  and  $n_t$ . On-Off SSK (OOSK) [8], which aims to provide optimal average bit error probability (ABEP), further aggravates the computational burden because of using different power levels in the signaling.

**Contributions:** We present a normalized compressed sensing based detection, NCS. We formulate the SSK-type demodulation criterion as a convex program via NCS. In addition, compared to the optimal maximum likelihood (ML) detection that exploits the conventional  $\ell_2$ -norm metric, NCS resorts to the  $\ell_1$ -norm metric and provides significant detection speedup by leveraging the inherent sparsity of SSK-type signaling. In particular, while ML detection demands  $O(N_r N_t^{n_t})$  complexity for GSSK, our proposed NCS achieves lower complexity  $O(n_t N_r N_t)$  at the cost of a certain level of detection accuracy degradation. Furthermore, despite numerous advantages over conventional MIMO systems, large-scale MIMO systems along its progress encounter major difficulties in the prohibitively high detection overhead. Our proposed NCS's detection offers the potentials to facilitate the large-scale MIMO systems (more details can be found in Sec. IV).

## II. SYSTEM OVERVIEW

**Problem Formulation:** We consider an uncoded spatial multiplexing system with  $N_t$  transmit antennas and  $N_r$  receive antennas. The baseband signal model is formulated as

$$\mathbf{y} = \sqrt{\rho} \mathbf{H} \mathbf{x} + \mathbf{n}, \quad (1)$$

where  $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$ ,  $\mathbf{x} \in \mathbb{R}^{N_t \times 1}$ ,  $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ , and  $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ , respectively, represent the received signal, transmitted sig-

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C.-M. Yu is with the Department of Electrical Engineering, National Taiwan University, and the Institute of Information Science, Academia Sinica.

S.-H. Hsieh is with the Institute of Information Science, Academia Sinica.

H.-W. Liang is with Department of Electrical Engineering, National Taiwan University.

C.-S. Lu is with the Institute of Information Science, Academia Sinica.

W.-H. Chung is with the Research Center for Information Technology Innovation, Academia Sinica (e-mail: whc@citi.sinica.edu.tw).

S.-Y. Kuo is with the Department of Electrical Engineering, National Taiwan University, and Beijing Jiaotong University.

S.-C. Pei is with the Graduate Institute of Communications Engineering, National Taiwan University.

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nal, flat-fading channel, and additive white Gaussian noise (AWGN). The  $\rho$  is defined as  $E_s/\mathbb{E}[\mathbf{x}^H\mathbf{x}]$ , where  $\mathbb{E}[\cdot]$  and  $E_s$  are the expectation operator and symbol energy, respectively. In GSSK,  $\mathbf{x}$  is a zero-one vector, where there are  $n_t$  ones (corresponding to activated transmit antennas) and  $N_t - n_t$  zeros (corresponding to idle transmit antennas). The  $n_t$  is fixed to be 1 in SSK and can be larger than 1 in GSSK. The antennas at the positions corresponding to the ones transmit signals with power level  $\sqrt{E_s}$ . Each element in  $\mathbf{H}$  is independent and identically complex Gaussian distributed  $\mathcal{CN}(0, 1)$  with mean 0 and variance 1, i.e., the real and imaginary parts of each element in  $\mathbf{H}$  is independent and identically Gaussian distributed  $\mathcal{N}(0, \frac{1}{2})$  with mean 0 and variance  $\frac{1}{2}$ . The channel information is assumed perfectly known at the receiver. The Additive White Gaussian Noise  $\mathbf{n}$  has zero-mean and covariance matrix  $N_0\mathbf{I}_{N_r}$ , where  $N_0$  and  $\mathbf{I}_{N_r}$  are the component-wise noise variance and  $N_r \times N_r$  identity matrix. We do not assume particular patterns of the transmitted symbol  $\mathbf{x}$ ; instead, it is drawn equally probably from the modulation alphabet (or the constellation set)  $\mathbb{A} = \{\mathbf{x}_1 \mathbf{x}_2 \cdots \mathbf{x}_{N_t}\}^T | \sum_{i=1}^{N_t} \mathbf{x}_i = n_t, \mathbf{x}_i \in \{0, 1\}\}$  with  $|\mathbb{A}| = 2^b$ , where  $b = \operatorname{argmin}_{\substack{2^b - \binom{N_t}{n_t} \\ \geq 0}} \left\{ 2^b - \binom{N_t}{n_t} \right\}$ , for  $b$  bits per transmission. The system model in this work has been commonly practiced as in [1], [5], [6], [7], [8].

The receiver aims to detect the antenna indices of signaling ones at the transmitter, which is termed as  $N_t$ -hypothesis detection problem.

**Maximum Likelihood (ML) Detection:** The ML detector attempts to solve the  $N_t$ -hypothesis problem by the  $\ell_2$ -norm criterion:

$$\hat{\mathbf{x}}_{ML} = \operatorname{argmin}_{\mathbf{x} \in \mathbb{A}} \|\mathbf{y} - \sqrt{E_s}\mathbf{H}\mathbf{x}\|_{\ell_2}^2. \quad (2)$$

To obtain the ML solution, the receiver searches over all the legal patterns in  $\mathbf{x}$ , i.e., all combinations with  $n_t$  nonzero elements, for the minimum distance between  $\mathbf{y}$  and  $\mathbf{H}\mathbf{x}$ .

**Compressed Sensing (CS):**  $\mathbf{x} \in \mathbb{R}^n$  is a  $s$ -sparse vector of length  $n$  with  $s < n$ . Here, “ $s$ -sparse” means there exist only  $s$  nonzero elements in  $\mathbf{x}$ . CS is formulated as  $\mathbf{y} = \Phi\mathbf{x}$ , where  $\mathbf{y} \in \mathbb{R}^m$  and  $\Phi \in \mathbb{R}^{m \times n}$ , with  $m < n$ , are called measurement vector and measurement matrix, respectively. In such formulation, provided that  $\Phi$  is a matrix satisfying the *restricted isometry property* (RIP) and  $m$  is greater than  $c_1 s \log \frac{n}{s}$  for some small constant  $c_1$ , then  $\mathbf{x}$  can be reconstructed with high probability by  $\ell_1$ -minimization as follows:

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{y}=\Phi\mathbf{x}} \|\mathbf{x}\|_{\ell_1}. \quad (3)$$

Note that RIP identifies the so-called *isometry constant*  $\delta_s$  of the measurement matrix  $\Phi$  as the smallest number such that  $(1 - \delta_s)\|\mathbf{x}\|_2^2 \leq \|\Phi\mathbf{x}\|_2^2 \leq (1 + \delta_s)\|\mathbf{x}\|_2^2$  holds for  $s$ -sparse vector  $\mathbf{x}$ . If the sampling matrix  $\Phi$  is designed properly to satisfy  $\delta_{c_2 s} < \theta$  for some constant  $c_2$ , then the perfect recovery of  $s$ -sparse vector can be achieved with very high probability if  $m = O(\frac{c_2 s}{\theta^2} \log \frac{n}{s})$  measurements are used [4].

The CS has a beneficial and tractable property where (3) is a convex program and can be easily solved by simple algorithms. It has been proven that the matrix whose elements are randomly sampled from  $\mathcal{N}(0, \frac{1}{m})$  satisfies the RIP with high probabilities. The RIP is to ensure that each pair of columns

of  $\Phi$  is orthogonal to each other with a high probability. In this sense, the matrix whose elements are sampled from  $\mathcal{N}(0, \sigma^2)$ ,  $\sigma^2 \geq 1$ , is highly likely to satisfy RIP.

The CS can be applied to not only  $s$ -sparse vectors but also *compressible vectors*. Conceptually, compressible vectors are vectors, with rapidly decaying sorted magnitudes. As those small-magnitude nonzero elements can be regarded as noise, CS can also be applied to noisy vector [2], [3].

### III. PROPOSED METHOD

The GSSK is considered below to better illustrate the flexibility of our proposed NCS detection. Note that our proposed detection is termed as NCS because a Normalization procedure needs to be performed before CS recovery. The algorithmic procedures of NCS is listed in Fig. 1.

**Algorithm:**  $\mathcal{J}=\text{NCS}(\mathbf{y}, \mathbf{H}, n_t)$

**Input:**  $\mathbf{y}$ : the received signal (measurement vector)

$\mathbf{H}$ : the channel information

$n_t$ : the number of activated transmit antennas

**Output:**  $\mathcal{J}$ : the set of column indices used at transmitter

1 for  $t = 1$  to  $N_t$

2 Let  $\mathbf{H}'_{-,t}$  be normalized  $t$ -th column  $\mathbf{H}_{-,t}$  of  $\mathbf{H}$

3  $\mathbf{H}' := [\mathbf{H}'_{-,1} \cdots \mathbf{H}'_{-,N_t}]$

4  $\hat{\mathbf{x}} := \text{CSRecovery}(\mathbf{y}, \mathbf{H}', n_t)$

5  $\mathcal{J} :=$  set of positions of the first  $n_t$  largest entries in  $\hat{\mathbf{x}}$

Fig. 1. NCS detection algorithm.

In essence, NCS performs CS recovery to accomplish SSK detection. There is a natural parameter mapping between SSK detection and CS, i.e.,  $N_t$ ,  $N_r$ , and  $n_t$  in SSK detection are mapped to  $n$ ,  $m$ , and  $s$  in CS, respectively.

The rationale behind the NCS design is that, in contrast to ML's  $\ell_2$  criterion where the information of known activated antennas is not fully exploited, the NCS algorithm takes the number of activated antennas into account explicitly to gain a significant decoding speedup. In particular, we formulate SSK detection as a CS-related  $\ell_1$  optimization problem. As CS offers a systematic way to recover  $\mathbf{x}$  by exploiting the property of  $n_t$  being much less than  $N_t$ ,  $\mathbf{x}$  can be considered sparse and this observation relates the SSK decoding to the CS recovery perfectly. Since CS offers an efficient way (e.g., greedy algorithm), instead of the exhaustive search-like  $\ell_2$ -norm based algorithm in ML detection, to recover  $\mathbf{x}$ , the detection time can be dramatically reduced.

Although applying existing CS recovery algorithms directly may solve the detection problem, it has unsatisfactory detection accuracy. Therefore, NCS is proposed to better solve for  $N_t$ -hypothesis detection problem. Specifically, in contrast to conventional CS that the sensing and recovery share the same measurement matrix, NCS allows different matrices in these two phases. As  $\mathbf{H}$  is determined by the channel condition and uncontrollable,  $\mathbf{H}'$  is obtained by simply normalizing each column vector  $\mathbf{H}_{-,i}$ ,  $1 \leq i \leq N_t$ , of  $\mathbf{H}$  individually (steps 1~3). The purpose of normalization is that the noise will not be over-amplified by the column vector with inadequately large elements. After this normalization pre-processing, CS recovery algorithm is conducted with  $\mathbf{H}'$  to derive  $\hat{\mathbf{x}}$  (the estimated  $\mathbf{x}$ , step 4). Here, the function,  $\text{CSRecovery}(\cdot)$ , can be

any off-the-shelf CS recovery algorithms (OMP [12] is used in our numerical simulations<sup>1</sup>). As the receiver has the prior knowledge of the energy of the activated transmit antennas in GSSK, only the first  $n_t$  largest elements in magnitude in  $\hat{\mathbf{x}}$  are set as 1 and the remaining are set as 0 (step 5). The set  $\mathcal{J}$  of positions of those 1's are considered as the set of indices of activated transmit antennas.

Two rationales in NCS are elaborated as follows. The first is the appropriateness of applying CS to  $N_t$ -hypothesis detection problem in which  $\mathbf{H}$  is a complex matrix. It is recalled that from Sec. II, the measurement matrix in CS is real-valued. Nevertheless, as  $\mathbf{x}$  is real-valued, which is the case of GSSK in our consideration, the imaginary parts of complex elements of  $\mathbf{H}$  can be considered as additional dimensions in the received signal  $\mathbf{y}$ . Consider  $\mathbf{y} = \mathbf{H}\mathbf{x}$  for explanation simplicity. The received signal  $\mathbf{y}$  can also be rewritten as:

$$\begin{aligned} & \begin{bmatrix} \mathbf{H}_{1,1}^r + j\mathbf{H}_{1,1}^i & \cdots & \mathbf{H}_{1,1}^r + j\mathbf{H}_{1,N_t}^i \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{N_r,1}^r + j\mathbf{H}_{N_r,1}^i & \cdots & \mathbf{H}_{N_r,1}^r + j\mathbf{H}_{N_r,N_t}^i \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{N_t} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{y}_1^r \\ \vdots \\ \mathbf{y}_{N_r}^r \end{bmatrix} + j \begin{bmatrix} \mathbf{y}_1^i \\ \vdots \\ \mathbf{y}_{N_r}^i \end{bmatrix} = \mathbf{H}^r \mathbf{x} + j\mathbf{H}^i \mathbf{x}, \end{aligned} \quad (4)$$

with  $\mathbf{H}_{s,t}^r$  and  $\mathbf{H}_{s,t}^i$  denoting real and imaginary parts of the  $t$ -th column of the element at the  $(s,t)$  position of  $\mathbf{H}$ , respectively, and with  $\mathbf{y}_t = \mathbf{y}_t^r + j\mathbf{y}_t^i$ ,  $1 \leq t \leq N_r$ , where  $\mathbf{y}_t^r$  and  $\mathbf{y}_t^i$  denote the real and imaginary parts of  $\mathbf{y}_t$ , respectively. Let  $\mathbf{y} = \mathbf{y}^r + j\mathbf{y}^i$ , where  $\mathbf{y}^r$  and  $\mathbf{y}^i$  are the real and imaginary parts of  $\mathbf{y}$ , respectively. We know that both real and imaginary parts of samples from  $\mathcal{CN}(0, \sigma^2)$  follow  $\mathcal{N}(0, \frac{\sigma^2}{2})$ . Recall that CS only requires the matrix elements in  $\mathbf{H}$  to follow Gaussian distribution with limited variance, as stated in Sec. II. The received signal can be divided into two parts,  $\mathbf{y}^r = \mathbf{H}^r \mathbf{x}$  and  $\mathbf{y}^i = \mathbf{H}^i \mathbf{x}$ , each of which satisfies the CS formulation, and  $\mathbf{H}^r$  and  $\mathbf{H}^i$  with  $\mathcal{N}(0, \frac{1}{2})$  can also satisfy RIP, as stated in Sec. II. Thus, equivalently the dimension of  $\mathbf{y}$  can be considered to be increased with the interpretation that the half of  $\mathbf{y}$  is generated by the  $\mathbf{H}^r$  and the other half is generated by  $\mathbf{H}^i$ .

The second point to be clarified is the usage of different matrices in sensing and recovery in NCS. As  $\sqrt{E_s}$  in (1) is constant, we omit it and consider

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (5)$$

for explanation simplicity. In other words, the received signal  $\mathbf{y}$  is generated via (5), which can be re-casted as:

$$\mathbf{y} = \mathbf{H}'\mathbf{C}\mathbf{x} + \mathbf{n}, \quad (6)$$

where  $\mathbf{H}'$  is of dimension  $N_r \times N_t$  and  $\mathbf{C} \in \mathbb{R}^{N_t \times N_t}$  is a diagonal matrix whose diagonal elements  $\mathbf{C}_{i,i}$  are the  $\ell_2$ -norm of  $\mathbf{H}_{-,i}$ , where  $\mathbf{H}_{-,i}$  denotes the  $i$ -th column of  $\mathbf{H}$ . Let  $\mathbf{x}' = \mathbf{C}\mathbf{x}$ . The Eq. (6) can be rewritten as

$$\mathbf{y} = \mathbf{H}'\mathbf{x}' + \mathbf{n}, \quad (7)$$

<sup>1</sup>OMP tries to find non-zero elements of  $\mathbf{x}$  in the noisy CS formulation  $\mathbf{y} = \Phi\mathbf{x} + \mathbf{e}$ , where  $\mathbf{e}$  is noise, by correlation between columns of  $\Phi$  and  $\mathbf{y}$ , i.e.,  $\Phi^T \mathbf{y} = \Phi^T \Phi \mathbf{x} + \mathbf{e}$ . When  $\Phi^T \Phi$  is nearly orthonormal, larger coefficients of  $\Phi^T \mathbf{y}$  are exactly corresponding to non-zero coefficients of  $\mathbf{x}$ . Without normalization, non-diagonal entries of  $\Phi^T \Phi$  are possibly larger than diagonal ones and OMP will not function normally.

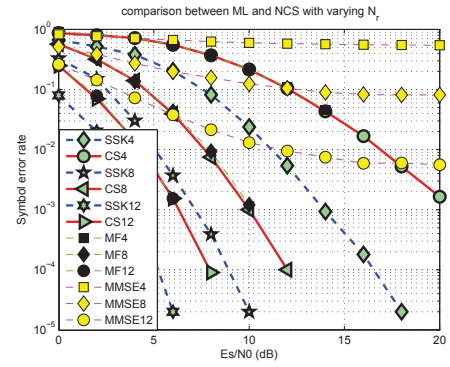


Fig. 2. The comparisons among ML, MMSE, MF, and NCS with varying  $N_r$ . SSK $t$ , MMSE $t$ , MF $t$ , and CS $t$  denote the ML, MMSE, MF, and NCS with  $N_r = t$ . The common settings include  $N_t = 256$ ,  $n_t = 1$ , each element of  $\mathbf{H}$  following  $\mathcal{CN}(0, 1)$ , and each element of  $\mathbf{n}$  following  $\mathcal{CN}(0, 1)$ .

where  $\mathbf{x}'$  is of dimension  $N_t \times 1$ . Observe that as  $\mathbf{C}$  is a diagonal matrix, the sets of positions of zeros and nonzeros in  $\mathbf{x}$  and  $\mathbf{x}'$  remain unchanged. The difference between  $\mathbf{x}$  and  $\mathbf{x}'$  is the magnitudes of those nonzero elements. Therefore, though the received signal  $\mathbf{y}$  is generated according to (5), the position set obtained by performing  $\hat{\mathbf{x}} := \text{CSRecovery}(\mathbf{y}, \mathbf{H}', n_t)$  (step 4), equivalently to performing

$$\hat{\mathbf{x}} := \underset{\mathbf{y}=\mathbf{H}'\mathbf{x}}{\operatorname{argmin}} \|\mathbf{x}\|_{\ell_1}, \quad (8)$$

is equivalent to the one obtained by performing

$$\hat{\mathbf{x}} := \underset{\mathbf{y}=\mathbf{H}\mathbf{x}}{\operatorname{argmin}} \|\mathbf{x}\|_{\ell_1}. \quad (9)$$

Hence, after the normalization and different matrices used in CS sensing and recovery, conventional CS algorithms are applicable to the  $N_t$ -hypothesis detection problem. Moreover, the normalization enables the extra advantage of avoiding unnecessary noise amplification, as stated above.

#### IV. PERFORMANCE EVALUATION

Here we present the symbol error rate (SER) performance of the considered detection algorithms. The ML detections for SSK and GSSK are implemented for comparisons. We also implement MMSE and MF (Matching Filter) [11] methods for comparison. We chose OMP [12] as an implementation of CSRecovery( $\cdot$ ) due to its implementation simplicity and performance efficiency while simplex method is another option to be applied to CS. Note that the computational load varies greatly with the CS recovery method used in the implementation. The CS complexity mentioned in this paper only applies for OMP. More sophisticated CS recovery methods are available to reduce the computation overhead or improve the detection accuracy.

**Detection Complexity:** By observing our algorithm in Fig. 1, it can be seen that steps 1~3 require  $O(N_r N_t)$  operations. As shown in [12], our choice of the implementation of CSRecovery( $\cdot$ ), OMP, needs  $O(n_t N_r N_t)$  operations. The step 5 requires  $O(N_t)$  operations because of searching the first  $n_t$  largest elements in a vector of length  $N_t$ . Overall, the detection complexity of NCS is  $O(n_t N_r N_t)$ . More specifically, NCS needs approximately  $n_t(N_t N_r + N_r n_t + N_r n_t^2 + n_t^3 + n_t^2)$  scalar addition and multiplication operations. To further verify the



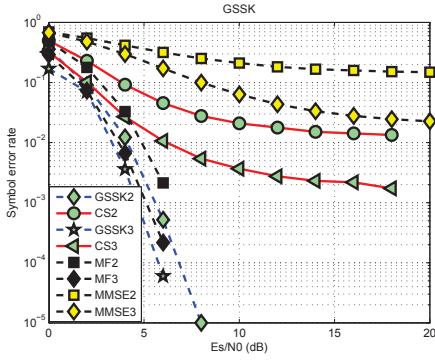


Fig. 3. The comparisons among ML, MMSE, MF, and NCS from the GSSK point of view. GSSK2, MMSE2, MF2, and CS2 denote the settings of ML, MMSE, MF, and NCS with  $N_r = 16$ ,  $N_t = 256$ ,  $n_t = 2$ . GSSK3, MMSE3, MF3, and CS3 denote the settings of ML, MMSE, MF, and NCS with  $N_r = 24$ ,  $N_t = 64$ ,  $n_t = 3$ . The common setting is that each element of  $\mathbf{H}$  follows  $\mathcal{CN}(0, 1)$ , and each element of  $\mathbf{n}$  follows  $\mathcal{CN}(0, 1)$ .

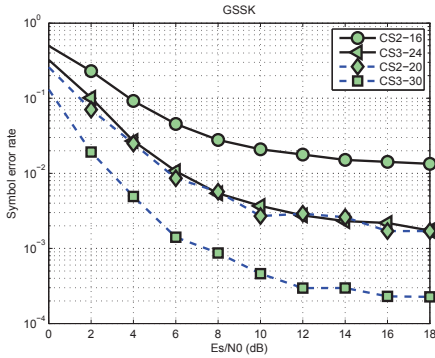


Fig. 4. The comparisons among different NCS's from the GSSK point of view. CS2-16 is with the setting of  $N_r = 16$ ,  $N_t = 256$ ,  $n_t = 2$ . CS2-20 is with the setting of  $N_r = 20$ ,  $N_t = 256$ ,  $n_t = 2$ . CS3-24 is with the setting of  $N_r = 24$ ,  $N_t = 64$ ,  $n_t = 3$ . CS3-30 is with the setting of  $N_r = 30$ ,  $N_t = 64$ ,  $n_t = 3$ .

detection speedup of our proposed NCS, a simple experiment with real site tests is conducted. On a laptop with INTEL i3 2.1 GHz CPU and 4GB RAM, performing ML detection on GSSK with  $N_t = 64$ ,  $N_r = 24$ , and  $n_t = 4$ , requires approximately 1 second per detection while performing NCS on the same setting requires only 0.002 seconds

**Numerical Results:** Figure 2 shows the comparisons among ML, MMSE [11], MF [11], and NCS with varying  $N_r$ . It can be observed that more receive antennas (larger  $N_r$ ) enable the narrower gap between ML and NCS. This can be attributed to the fact that a threshold number of measurements, as shown in Sec. II, are needed to accomplish the required accurate recovery.

The results of GSSK are depicted in Fig. 3. It is observed that the error floor occurs in the proposed NCS detection. This is due to the nature of compressed sensing, which serves as the fundamental mechanism of NCS. The theory of compressed sensing states that in the formulation of  $y = Ax + e$ , where  $A$  is an Gaussian measurement matrix of dimension  $m \times n$ ,  $x$  is the original signal of dimension  $n$  with  $s$  nonzero elements,  $e$  is the Gaussian noise, and  $y$  is the measurement of dimension  $m$ ,  $x$  can be accurately recovered from  $y$  provided  $m \geq cs \log(\frac{n}{s})$  for some constant  $c$ . In compressed sensing, the quality of

recovery result is primarily dominated by the number  $m$  of measurements and can be improved once the number  $m$  of measurements is increased. It turns out that in GSSK detection, the error floor in NCS can be improved once  $N_r$  is increased. The observation of Fig. 4 also confirms such assertion.

**Discussion:** As ML is highly complicated in large-scale high-rate MIMO systems, our proposed NCS detection algorithm has great potential in such MIMO systems. In particular, in a GSSK modulated MIMO system with  $N_r = 256$ ,  $N_t = 2048$ , and  $n_t = 16$ , the attainable symbol rate is approximately  $\lfloor \log \binom{2048}{16} \rfloor \approx 131$  bits<sup>2</sup>. Nonetheless, ML detection leads to the running time explosion due to its required  $256 \cdot 2048^{16}$  operations. Since approximately merely  $256 \cdot 2048 \cdot 16 = 8388608$  operations are sufficient in NCS, the proposed NCS serves as a lightweight and promising alternative for GSSK demodulation. In particular, in the above case, NCS can successfully detect the antenna indices as long as the  $E_s/N_0$  is more than 6dB.

## V. CONCLUSION

The proposed NCS algorithm exploits the sparsity in SSK signaling and uses  $\ell_1$ -norm metric as opposed to the  $\ell_2$ -norm metric in conventional detector. Our proposed NCS provides the advantage of a convex formulation to the SSK-type demodulations. The complexity analyses and simulations show that NCS offers significant detection speedup over the optimal ML detection at slight performance degradations.

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<sup>2</sup>It is noted that such large scale system may not be common in the currently adopted MIMO systems. This example is examined for two purposes: 1) to investigate the proposed NCS applied to the future large system; 2) to demonstrate the performance gap between ML and NCS.