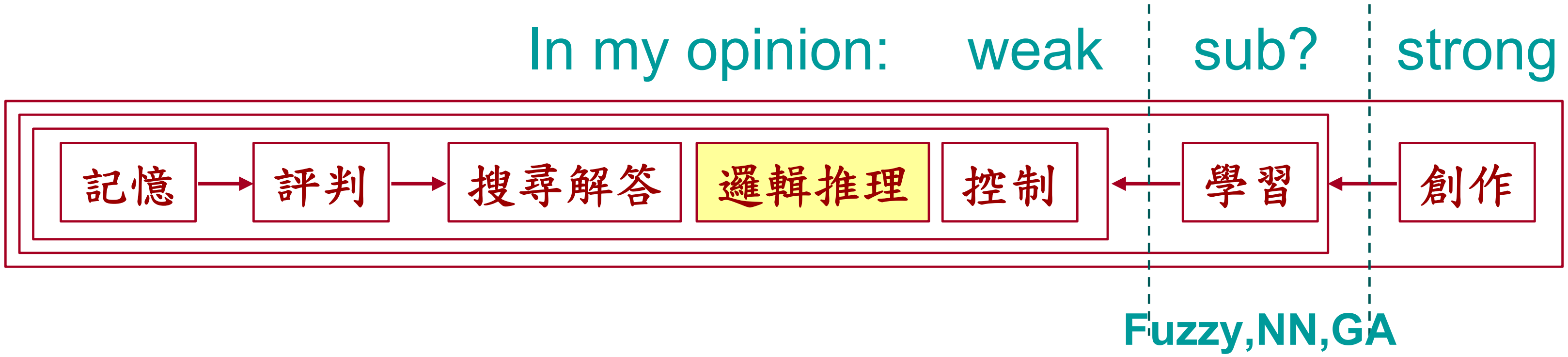
Chapter 7

Propositional and Predicate Logic

What is Artificial Intelligence?

- A more difficult question is: What is intelligence?
- This question has puzzled philosophers, biologists and psychologists for centuries.
- Artificial Intelligence is easier to define, although there is no standard, accepted definition.

星艦戰將與
蝸蟲的學習



Chapter 7 Contents

- What is Logic?
- Logical Operators
- Translating between English and Logic
- Truth Tables
- Complex Truth Tables
- Tautology
- Equivalence
- Propositional Logic
- Deduction
- Predicate Calculus
- Quantifiers ∀ and ∃
- Properties of logical systems
- Abduction and inductive reasoning
- (Modal logic)

What is Logic?

- **Reasoning about the validity of arguments.**
- **An argument is valid if its conclusions follow logically from its premises – even if the argument doesn't actually reflect the real world:**
  - All lemons are blue
  - Mary is a lemon
  - Therefore, Mary is blue.

Logical Operators

- And $\wedge$
- Or $\vee$
- Not $\neg$
- Implies $\rightarrow$ (if… then…)
- Iff $\leftrightarrow$ (if and only if)

Translating between English and Logic

- **Facts and rules need to be translated into logical notation.**
- **For example:**
  - It is Raining and it is Thursday:
  - $R \wedge T$
  - R means “It is Raining”, T means “it is Thursday”.

Translating between English and Logic

- **More complex sentences need predicates. E.g.:**
  - It is raining in New York:
  - R(N)
  - Could also be written N(R), or even just R.
- **It is important to select the correct level of detail for the concepts you want to reason about.**

Truth Tables

- Tables that show truth values for all possible inputs to a logical operator.
- For example:

A	B	$A \wedge B$
false	false	false
false	true	false
true	false	false
true	true	true

A	B	$A \vee B$
false	false	false
false	true	true
true	false	true
true	true	true

- A truth table shows the semantics of a logical operator.

Complex Truth Tables

- **We can produce truth tables for complex logical expressions, which show the overall value of the expression for all possible combinations of variables:**

A	B	C	$A \wedge (B \vee C)$
false	false	false	false
false	false	true	false
false	true	false	false
false	true	true	false
true	false	false	false
true	false	true	true
true	true	false	true
true	true	true	true

Tautology (恆真式)

- The expression $A \vee \neg A$ is a tautology.
- This means it is always true, regardless of the value of A.
- A is a tautology: this is written
  $\models A$

A	$A \vee \neg A$
false	true
true	true

  - **A tautology is true under any interpretation.**
  - **An expression which is false under any interpretation is contradictory.**
- 一切數學證明的動作來自: $\models (P \wedge (P \rightarrow Q) \rightarrow Q)$
  或者強調那是動作的結果: $P \wedge (P \rightarrow Q) \vdash Q$

Equivalence

- Two expressions are equivalent if they always have the same logical value under any interpretation:
  - $A \wedge B \equiv B \wedge A$
- Equivalences can be proven by examining truth tables.

Some Useful Equivalences

- $A \vee A \equiv A$
- $A \wedge A \equiv A$
- $A \wedge (B \wedge C) \equiv (A \wedge B) \wedge C$
- $A \vee (B \vee C) \equiv (A \vee B) \vee C$
- $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
- $A \wedge (A \vee B) \equiv A$
- $A \vee (A \wedge B) \equiv A$

- $A \wedge \text{true} \equiv A$ $\quad$ $A \wedge \text{false} \equiv \text{false}$
- $A \vee \text{true} \equiv \text{true}$ $\quad$ $A \vee \text{false} \equiv A$

Propositional Logic（命題邏輯）

- Propositional logic is a **logical system.**
- It deals with propositions.
  - Inference (what results from assumptions?)
  - Reasoning (is it true or not?)
- Propositional Calculus is the language we use to reason about propositional logic.
- A sentence in propositional logic is called a well-formed formula (wff).
  - Wff: English → logic sentence

Propositional Logic

- **The following are wff's:**
- P, Q, R...
- true, false
- (A)
- ¬A
- A ∧ B
- A ∨ B
- A → B
- A ↔ B

定義 **P→Q** ≡ ¬ P V Q （真值表比對）
e.g.

Tall ^ Strong → Ball_Player
≡ ¬ (Tall ^ Strong) V Ball_Player
≡ ¬ Tall V ¬ Strong V Ball_Player

Recall:

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- Deduction
- Predicate Calculus
- Quantifiers $\forall$ and $\exists$
- Properties of logical systems
- Abduction and inductive reasoning
- (Modal logic)

Deduction

- **The process of deriving a conclusion from a set of assumptions.**
- **Use a set of rules, such as:**

$$\frac{\mathbf{A} \qquad\qquad \mathbf{A \rightarrow B}}{\mathbf{B}}$$

7.11(p.191 ~ p.195)

(modus ponens… 拉丁文:推論法)

- **If we deduce a conclusion C from a set of assumptions, we write:**
- $\{A_1, A_2, \ldots, A_n\} \vdash C$

Deduction - Example

$\neg A \quad \neg A \rightarrow B$	assumptions
$B \quad \neg B$	modus ponens
$B \quad B \rightarrow \perp$	rewriting $\neg B$
$\perp$	modus ponens
A	reductio ad absurdum
$\neg B \rightarrow A$	$\rightarrow$ introduction
$(\neg A \rightarrow B) \rightarrow (\neg B \rightarrow A)$	$\rightarrow$ introduction

(1)以推導方式取代真值表驗證，更簡單而有意義；
(2)但盲目的推導方法類似盲目搜尋，在**chap 8** 有改良的方法。

Predicate Calculus（述語推算）

- **Predicate Calculus extends the syntax of propositional calculus with predicates and quantifiers:**
  - P(X) – P is a predicate.
- **First Order Predicate Calculus (FOPC) allows predicates to apply to objects or terms, but not functions or predicates.**

Quantifiers ∀ and ∃

- ∀ - For all:
  - ∀xP(x) is read "For all x'es, P (x) is true".
- ∃ - There Exists:
  - ∃x P(x) is read "there exists an x such that P(x) is true".
- Relationship between the quantifiers:
  - $\exists x\ P(x) \equiv \neg(\forall x)\neg P(x)$
  - "If There exists an x for which P holds, then it is not true that for all x P does not hold".

$$\boxed{\exists x\ \mathrm{Like}(x, \mathrm{War}) \equiv \neg(\forall x)\ \neg \mathrm{Like}(x, \mathrm{War})}$$

Deduction over FOPC --- Search

- Dog(X) ^ Meets(X,Y)^Dislikes(X,Y) → Barks_at(X,Y)
- Close_to(Z, DormG) → Meets(Snoopy, Z)
- Man(W) → Dislikes(Snoopy, W)
- Man(John), Dog(Snoopy), Close_to(John,DormG)

{John/W}

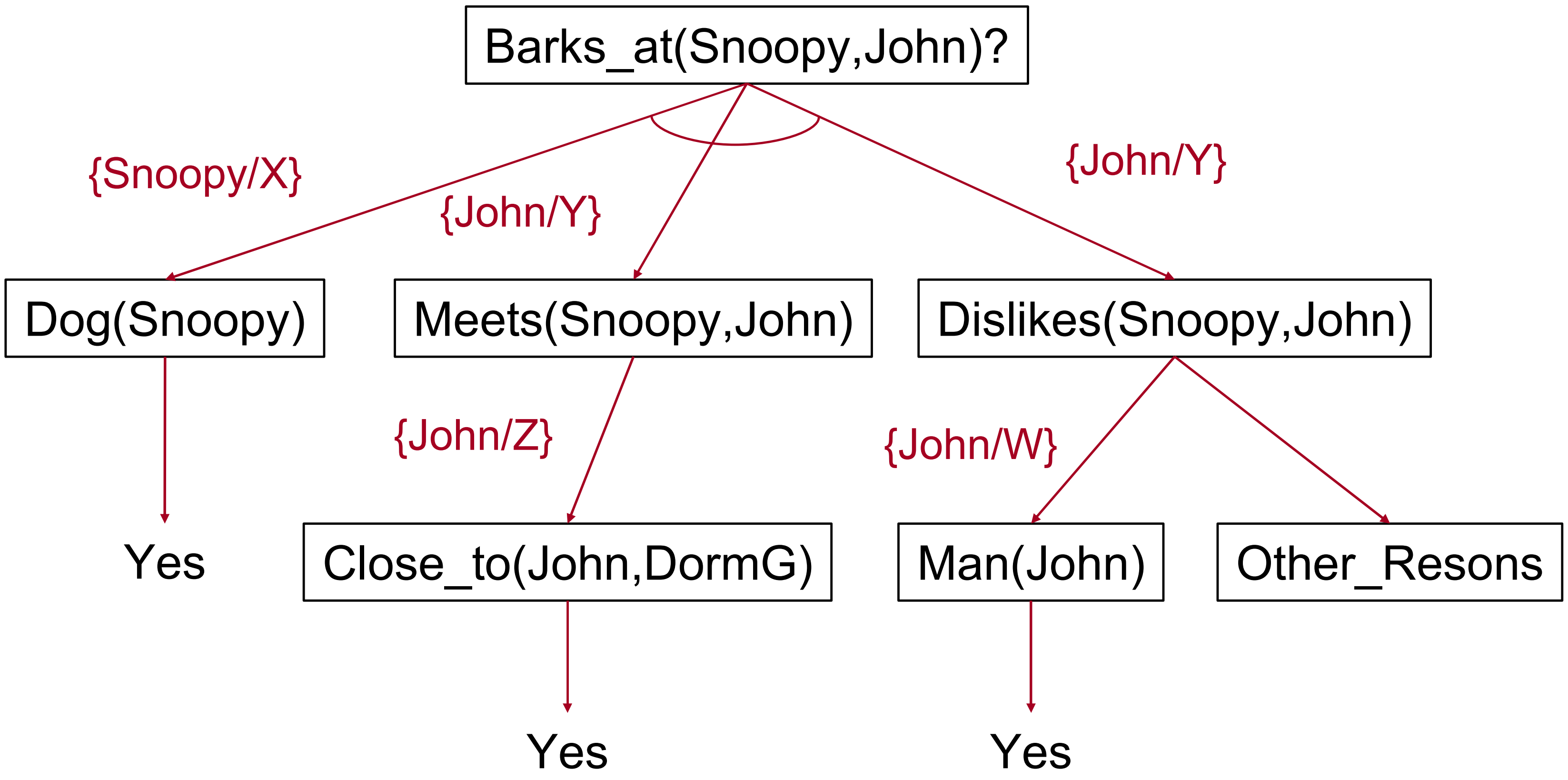
- Dog(X) ^ Meets(X,Y)^Dislikes(X,Y) → Barks_at(X,Y)
- Close_to(Z, DormG) → Meets(Snoopy, Z)
- Dislikes(Snoopy, John)
- Dog(Snoopy), Close_to(John,DormG)

??

Barks_at(Snoopy,John)

Recall:

Deduction over FOPC --- Goal Tree



Properties of Logical Systems

- **Soundness(可靠): Is every theorem valid?**
- **Completeness(週延): Is every tautology a theorem?**
- **Decidability(可推導): Does an algorithm exist that will determine if a wff is valid?**
- **Monotonicity(不受破壞): Can a valid logical proof be made invalid by adding additional premises or assumptions?**

Abduction and Inductive Reasoning

- Abduction:

$$\frac{B \qquad\qquad A \rightarrow B}{A}$$

過度演繹

- **Not** logically valid, BUT can still be useful.
- In fact, it models the way humans reason all the time:
  - Every non-flying bird I've seen before has been a penguin; hence that non-flying bird must be a penguin.
- Not valid reasoning, but likely to work in many situations.

Skip:

Modal logic

- Modal logic is a higher order logic.
- Allows us to reason about **certainties**, and possible worlds.
- If a statement A is contingent then we say that A is possibly true, which is written:

  $\Diamond A$

- If A is non-contingent, then it is necessarily true, which is written:

  $\Box A$

cf. “fuzzy logic” … to appear later