

# Discrete Mathematics

## Quiz 1

April 12, 2011

1. Let  $G$  be an undirected graph without multiple edges. Let  $V_G$  and  $E_G$  be the vertex set and edge set.
  - (a) Suppose  $|V_G| = n$ . What is the range of  $|E_G|$  (that is, how small and how large can  $|E_G|$  get)?
  - (b) In addition, suppose that  $G$  is a connected graph. What is the range of  $|E_G|$ ?
  - (c) In addition, suppose that  $G$  is a tree. What is the range of  $|E_G|$ ?
2. Let  $K_n$  be the complete graph of  $n$  vertices.
  - (a) Does there exist an Euler circuit in  $K_n$  for each  $n$ ? Prove or disprove.
  - (b) Does there exist a Hamilton circuit in  $K_n$  for each  $n$ ? Prove or disprove.
3. Let  $G$  be a directed graph with multiple edges. Define a relation  $R \subseteq V_G \times V_G$  such that  $x R y$  if there is a path from vertex  $x$  to vertex  $y$  in  $G$ .
  - (a) Is  $R$  always a reflexive relation? Prove or disprove.
  - (b) Is  $R$  always a symmetric relation? Prove or disprove.
  - (c) Is  $R$  always a transitive relation? Prove or disprove.
4. 讓命題變數  $p =$  「台灣宣告獨立」， $q =$  「台灣遭受攻擊」。
  - (a) 口號「不獨則不武」所表達的命題敘述為何？
  - (b) 口號「不武則不獨」所表達的命題敘述為何？
  - (c) 以上兩項命題敘述分別為恆真敘述 (tautology) 嗎？
  - (d) 「不武則不獨」等同於「不獨則不武」嗎？
5. Exercise 1.3.19