Depth-First Iterative-Deepening: An Optimal Admissible Tree Search

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Abstract

- **The complexities of various search algorithms are considered in terms of time, space, and cost of the solution paths.**
  - **Brute-force search**
    - ▷ ***Breadth-first search (BFS)***
    - ▷ ***Depth-first search (DFS)***
    - ▷ ***Depth-first Iterative-deepening (DFID)***
    - ▷ ***Bi-directional search***
  - **Heuristic search: best-first search**
    - ▷ A^*
    - ▷ IDA^*
- **The issue of storing information in DISK instead of main memory.**
- **Solving 15-puzzle.**

Definitions

- **Node branching factor b: the number of different new states generated from a state.**
  - **Average node branching factor.**
  - **Assumed to be a constant here.**
- **Edge branching factor e: the number of possible new, maybe duplicated, states generated from a state.**
  - **Average node branching factor.**
  - **Assumed to be a constant here.**
- **Depth of a solution d: the shortest length from the initial state to one of the goal states**
  - **The depth of the root is 0.**
- **A search program finds a goal state starting from the initial state by exploring states in the state space.**
  - **Brute-force search**
  - **Heuristic search**

Brute-force search

- **A brute-force search is a search algorithm that uses information about**
  - **the initial state,**
  - **operators on finding the states adjacent to a state,**
  - **and a test function whether a goal is reached.**
- **A "pure" brute-force search program.**
  - **A state maybe re-visited many times.**
- **An "intelligent" brute-force search algorithm.**
  - **Make sure a state will be visited a limited number of times.**
    - ▷ ***Make sure a state will be eventually visited.***

A “pure” brute-force search

- A “pure” brute-force search is a brute-force search algorithm that does not care whether a state has been visited before or not.
- Algorithm Brute-force(N_0)
  {∗ do brute-force search from the starting state N_0 ∗}
  - current $\leftarrow N_0$
  - While true do
    - ▷ *If current is a goal,*
      then return success
    - ▷ *current $\leftarrow$ a state that can reach current in one step*
- Comments
  - Very easy to code and use very little memory.
  - May take infinite time because there is no guarantee that a state will be eventually visited.

Intelligent brute-force search

- **An "intelligent" brute-force search algorithm.**
  - **Assume S is the set of all possible states**
  - **Use a systematic way to examine each state in S one by one so that**
    - ▷ ***A state is not examined too many times — does not have too many duplications.***
    - ▷ ***It is efficient to find an unvisited state in S.***
- **Need to know whether a state has been visited before efficiently.**
- **Some notable algorithms.**
  - **Breadth-first search (BFS).**
  - **Depth-first search (DFS) and its variations.**
  - **Depth-first Iterative deepening (DFID).**
  - **Bi-directional search.**

Breadth-first search (BFS)

- $deeper(N)$: **gives the set of all possible states that can be reached from the state N.**
  - **It takes at least $O(e)$ time to compute $deeper(N)$.**
  - **The number of distinct elements in $deeper(N)$ is b.**
- **Algorithm BFS(N_0)** **{$*$ do BFS from the starting state N_0 $*$}**
  - **If the starting state N_0 is a goal,**
    then return success
  - **Initialize a Queue Q**
  - **Add N_0 to Q**
  - **While Q is not empty do**
    - ▷ ***Remove a state N from Q***
    - ▷ ***If one of the states in $deeper(N)$ is goal,***
      then return success
    - ▷ ***Add states in $deeper(N)$ that have not been visited before to Q;***
    - ▷ ***Mark these newly added states as visited;***
      if there are duplications in $deeper(N)$, add only once;
  - **Return fail**

BFS: analysis

- **Space complexity:**
  - $O(b^d)$
    - ▷ ***The average number of distinct elements at depth*** d ***is*** b^d***.***
    - ▷ ***We need to store all distinct elements at depth*** d ***in the Queue.***
    - ▷ ***We need to keep a record on visited nodes in order not to re-visit them.***
- **Time complexity:**
  - $1 * e + b * e + b^2 * e + b^3 * e + \cdots + b^{d-1} * e = (b^d - 1) * e/(b-1) = O(b^{d-1} * e)$, **if** b **is a constant.**
    - ▷ ***For each element*** N ***in the Queue, it takes at least*** $O(e)$ ***time to find*** $deeper(N)$***.***
    - ▷ ***It is always true that*** $e \geq b$***.***

BFS: comments

- **Always finds an optimal solution, i.e., one with the smallest possible depth d.**
  - **Do not need to worry about falling into loops.**
- **Most critical drawback: huge space requirement.**
  - **It is tolerable for an algorithm to be 100 times slower, but not so for one that is 100 times larger.**

BFS: ideas when there is little memory

- **What can be done when you do not have enough main memory?**
  - **DISK**
    - ▷ *Store states that has been visited before into DISK and maintain them as sorted.*
    - ▷ *Store the QUEUE into DISK.*
  - **Memory: buffers**
    - ▷ *Most recently visited nodes.*
    - ▷ *Candidates of possible newly explored nodes.*
  - **Merge the buffer of visited nodes with the one in DISK when memory is full.**
    - ▷ *We only need to know when a newly explored node has been visited or not when it is about to be removed from the QUEUE.*
    - ▷ *The decision of whether it has been visited or not can be **delayed**.*
  - **Append the buffer of newly explored nodes to the QUEUE the DISK when memory is full or it is empty.**

BFS: disk based

- Algorithm BFS$_{disk}$(N_0)
  {$*$ do disk based BFS from the starting state N_0 $*$}
  - If the starting state N_0 is a goal, then return success
  - Initialize a Queue Q_d using DISK
  - Initialize a buffer Q_m of potential states to visit using main memory
  - Initialize a sorted list V_d of visited nodes using DISK
  - Initialize a buffer V_m of visited nodes using main memory
  - Add N_0 to Q_d
  - While Q_d and Q_m are not both empty do
    - ▷ *If Q_d is empty, then { Sort Q_m; Write Q_m to Q_d; Empty Q_m }*
    - ▷ *Remove a state N from Q_d*
    - ▷ *Add N to V_m*
    - ▷ *If V_m is full, then { Sort V_m; Merge V_m into V_d; Empty V_m }*
    - ▷ *If one of the states in $deeper(N)$ is goal, then return success*
    - ▷ *Add unvisited states in $deeper(N)$ to Q_m; Mark them as visited;*
    - ▷ *If Q_m is full, then {*
      Sort Q_m;
      Add states in Q_m that are not in V_d and V_m to Q_d;
      Empty Q_m }
  - Return fail

Disk based algorithms (1/3)

- **When data cannot be loaded into the memory, you need to re-invent algorithms even for tasks that may look simple.**
  - **Batched processing.**
    - ▷ ***Accumulate tasks and then try to perform these tasks when they need to.***
    - ▷ ***Combine tasks into one to save disk I/O time.***
    - ▷ ***Order disk accessing patterns.***
- **Main ideas:**
  - **When two files are sorted, it is cost effective to compare the difference of them.**
  - **It is not too slow to read all records of a large file in sequence.**
  - **It is very slow and cost a lot to read every record in a large file in a random order.**

Disk based algorithms (2/3)

- **Implementation of the QUEUE.**
  - **QUEUE can be stored in one disk file.**
  - **Newly explored ones are appended at the end of the file.**
  - **Retrieve the one at the head of the file.**
- **A newly explored node will be explored after the current QUEUE is empty.**

Disk based algorithms (3/3)

- **How to find out a list of newly explored nodes have been visited or not?**
  - **Maintain the list of visited nodes on DISK sorted.**
    - ▷ *When the member buffer is full, sort it.*
    - ▷ *Merge the sorted list of newly visited nodes in buffer into the one stored on DISK.*
  - **We can easily compare two sorted lists and find out the intersection or difference of the two.**
    - ▷ *We can easily remove the ones that are already visited before once Q_m is sorted.*
    - ▷ *To revert items in Q_m back to its the original BFS order, which is needed for persevering the BFS search order, we need to sort again using the original BFS ordering.*
- **Why we can delay the decision of whether a newly explored node has been visited or not?**
  - **We only need to know when a newly explored node has been visited or not when it is about to be removed from the QUEUE.**
  - **The decision of whether it has been visited or not can be delayed.**

Depth-first search (DFS)

- $next(current, N)$: **returns the state next to the state "$current$" in $deeper(N)$.**
  - **Assume states in $deeper(N)$ are given a linear order with dummy first and last elements both being $null$, and assume $current \in deeper(N)$.**
  - **Assume we can efficiently generate $next(current, N)$ based on "$current$" and N.**
- **Algorithm DFS(N_0)** {∗ do DFS from the starting state N_0 ∗}
  - **Initialize a Stack S**
  - **Push $(null, N_0)$ to S**
  - **While S is not empty do**
    - ▷ ***Pop*** $(current, N)$ ***from*** S
    - ▷ $R \leftarrow next(current, N)$
    - ▷ ***If*** R ***is a goal, then return success***
    - ▷ ***If*** R ***is*** $null$***, then continue*** {∗ *searched all children of* N ∗}
    - ▷ ***Push*** (R, N) ***to*** S
    - ▷ ***If*** R ***is already in*** S***, then continue*** {∗ *to avoid loops* ∗}
    - ▷ ***Push*** $(null, R)$ ***to*** S {∗ *search deeper* ∗}
    - ▷ ***Can introduce some cut-off depth here in order not to go too deep***
  - **Return fail**

DFS: analysis

- **Time complexity:**
  - $O(e^d)$
    - ▷ ***The number of possible branches at depth d is e^d.***
- **Space complexity:**
  - $O(d)$
    - ▷ ***Only need to store the current path in the Stack.***
- **Comments:**
  - **Without a good cut-off depth, it may not be able to find a solution in time.**
  - **May not find an optimal solution at all.**
  - **Heavily depends on the move ordering.**
    - ▷ ***Which one to search first when you have multiple choices for your next move?***
  - **A node can be searched many times.**
    - ▷ ***Need to do something, e.g., hashing, to avoid researching too much.***
    - ▷ ***Need to balance the effort to memorize and the effort to research.***
  - **Most critical drawback: huge and unpredictable time complexity.**

DFS: when there is little memory

- **Need to have a hash table to store the set of visited nodes in order not to visit a node too many times.**
  - **We need to decide instantly whether a node is visited or not.**
  - **The decision of whether a node is visited or not cannot be delayed.**
    - ▷ ***Batch processing is not working here.***
    - ▷ ***It takes too much time to handle a disk based hash table.***
- **Use data compression and/or bit-operation techniques to store as many visited nodes as possible.**
  - **Some nodes maybe visit again and again.**
  - **Need a good heuristic to store the most frequently visited nodes.**
    - ▷ ***Avoid swapping too often.***

DFS with a depth limit

- **Do DFS from the starting state N_0 without exceeding a given depth $limit$.**
  - **length(x, y): the number of edges between a shortest path from the node x and the node y.**
  - **Depth of a node x in a tree = length$(root, x)$.**
- **Algorithm DFS$_{depth}$($N_0, limit$)**
  - **Initialize a Stack S**
  - **Push $(null, N_0)$ to S where N_0 is the initial state**
  - **While S is not empty do**
    - ▷ ***Pop*** $(current, N)$ ***from*** S
    - ▷ $R \leftarrow next(current, N)$
    - ▷ ***If*** R ***is a goal, then return success***
    - ▷ ***If*** R ***is*** $null$***, then continue*** **{∗ *searched all children of* N ∗}**
    - ▷ ***If length***$(N_0, R) > limit$***, then continue*** **{∗ *cut off* ∗}**
    - ▷ ***Push*** (R, N) ***to*** S
    - ▷ ***If*** R ***is already in*** S***, then continue*** **{∗ *to avoid loops* ∗}**
    - ▷ ***Push*** $(null, R)$ ***to*** S **{∗ *search deeper* ∗}**
  - **Return fail**

Depth-first iterative-deepening (DFID)

- **DFS**$_{depth}(N, limit)$**: DFS from the starting state** N **and with a depth cut off at the depth** $limit$
- **Algorithm DFID(**N_0**,**cut_off_depth**) {**$*$ **do DFID from the starting state** N_0 **with a depth limit** cut_off_depth $*$**}**
  - $limit \leftarrow 0$
  - **While** $limit < cut\_off\_depth$ **do**
    - ▷ ***If DFS***$_{depth}(N_0, limit)$ ***finds a goal state*** g***, then return*** g ***as the found goal state***
    - ▷ $limit \leftarrow limit + 1$
  - **Return fail**
- **Space complexity:**
  - $O(d)$

Time complexity of DFID (1/2)

- **The branches at depth i are generated $d-i+1$ times.**
  - **There are e^i branches at depth i.**
- **Total number of branches visited $M(e,d)$ is**

$$
\begin{aligned}
&(d+1)e^0 + de^1 + (d-1)e^2 + \cdots + 2e^{d-1} + e^d \\
&= e^d(1 + 2e^{-1} + 3e^{-2} + \cdots + (d+1)e^{-d}) \\
&\leq e^d(1-1/e)^{-2} \text{ if } e > 1
\end{aligned}
$$

- **Analysis:**
  - ▷ $(1-x)^{-2} = 1/(1-2x+x^2) = 1 + 2x + 3x^2 + \cdots + kx^{k-1} + (k+1)x^k - kx^{k+1}$.
  - ▷ *Hence* $1 + 2x + 3x^2 + \cdots + kx^{k-1} \leq (1-x)^{-2}$, *if* $|x| < 1$.
  - ▷ *Since* $|x| < 1$,

$$\lim_{k\to\infty}((k+1)x^k - kx^{k+1}) = 0.$$

  - ▷ *If k is large enough and* $|x| < 1$, *then* $(1-x)^{-2} \approx 1 + 2x + 3x^2 + \cdots + kx^{k-1}$.

Time complexity of DFID (2/2)

- Let $M(e,d)$ be the total number of branches visited by DFID with an edge branching factor of e and depth d.
- Examples:
  - When $e = 2$, $M(e,d) \leq 4e^d$.
  - When $e = 3$, $M(e,d) \leq 9/4e^d$.
  - When $e = 4$, $M(e,d) \leq 16/9e^d$.
  - When $e = 5$, $M(e,d) \leq 25/16e^d$.
  - $M(e,d) = O(e^d)$ with a small constant factor.

DFID: comments

- **No need to worry about a good cut-off depth as in DFS.**
- **Still need a mechanism to decide instantly whether a node has been visited before or not.**
- **Good for a tournament situation where each move needs to be made in a limited amount of time.**
- **Q:**
  - *Does DFID always find an optimal solution?*
  - *How about BFID?*

DFS with depth limit and direction (1/2)

- **Two refined service routines when direction of the search is considered:**
  - **DFS**$_{dir}(B, G, successor, i)$: **DFS with the set of starting states** B, **goal states** G, $successor$ **function and depth limit** i.
  - $next_{dir}(current, successor, N)$: **returns the state next to the state "**$current$**" in** $successor(N)$.
- **In the above two routines:**
  - $successor$ **is** $deeper$ **for forward searching**
  - $successor$ **is** $prev$ **for backward searching**
- **Note:**
  - **Given a state** N, $prev(N)$ **gives all states that can reach** N **in one step.**
  - **Given a state** N, $deeper(N)$ **gives the set of all possible states that can be reached from the state** N **in one step.**

DFS with depth limit and direction (2/2)

- **DFS$_{dir}(B, G, successor, i)$: DFS with the set of starting states B, goal states G, $successor$ function and depth limit i.**
- **Algorithm DFS$_{dir}(B, G, successor, limit)$**
  - **Initialize a Stack S**
  - **For each possible starting state t in B do**
    - ▷ ***Push*** $(null, t)$ ***to*** S
  - **While S is not empty do**
    - ▷ ***Pop*** $(current, N)$ ***from*** S
    - ▷ $R \leftarrow next_{dir}(current, successor, N)$
    - ▷ ***If*** R ***is a goal in*** G***, then return success***
    - ▷ ***If*** R ***is*** $null$***, then continue*** **{∗ *searched all children of* N ∗}**
    - ▷ ***If length***$(B, R) > limit$***, then continue*** **{∗ *cut off* ∗}**
    - ▷ ***Push*** (R, N) ***to*** S
    - ▷ ***If*** R ***is already in*** S***, then continue*** **{∗ *to avoid loops* ∗}**
    - ▷ ***Push*** $(null, R)$ ***to*** S **{∗ *search deeper* ∗}**
  - **Return fail**
- **Note length(B, x) is the length of a shortest path between the state x and a state in B.**

Bi-directional search

- **Combined with iterative-deepening.**
- **DFS**$_{dir}(B, G, successor, i)$**: DFS with the set of starting states** B**, goal states** G**,** $successor$ **function and depth limit** i**.**
  - $successor$ **is** $deeper$ **for forward searching**
  - $successor$ **is** $prev$ **for backward searching**
    - ▷ ***Given a state*** S_i, $prev(S_i)$ ***gives all states that can reach*** S_i ***in one step.***
- **Algorithm BDS(**N_0,cut_off_depth**)**
  - $limit \leftarrow 0$
  - **while** $limit < cut\_off\_depth$ **do**
    - ▷ ***if DFS***$_{dir}(\{N_0\}, G, deeper, limit)$ ***returns success,*** ***then return success*** **{∗ *forward searching* ∗}** ***else store all states at depth*** $= limit$ ***in an area*** H
    - ▷ ***if DFS***$_{dir}(G, H, prev, limit)$ ***returns success,*** ***then return success*** **{∗ *backward searching* ∗}**
    - ▷ ***if DFS***$_{dir}(G, H, prev, limit + 1)$ ***returns success,*** ***then return success*** **{∗ *in case the optimal solution is odd-lengthed* ∗}**
    - ▷ $limit \leftarrow limit + 1$
  - **return fail**
- **Backward searching at depth** $= limit + 1$ **is needed to find odd-lengthed optimal solutions.**

Bi-directional search: analysis

- **Time complexity:**
  - $O(e^{d/2})$
- **Space complexity:**
  - $O(e^{d/2})$: **needed to store the half-way meeting points** H**.**
- **Comments:**
  - **Run well in practice.**
  - **Depth of the solution is expected to be the same for a normal uni-directional search, however the number of nodes visited is greatly reduced.**
  - **Pay the price of storing solutions at half depth.**
  - **Need to know how to enumerate the set of goals.**
  - **Trade off between time and space.**
    - ▷ ***What can be stored on DISK?***
    - ▷ ***What operations can be batched?***
  - **Q:**
    - ▷ ***How about using BFS in forward searching?***
    - ▷ ***How about using BFS in backward searching?***
    - ▷ ***How about using BFS in both directions?***

Heuristic search

- **Heuristics**: criteria, methods, or principles for deciding which among several alternative courses of actions promises to be the most effective in order to achieve some goal [Judea Pearl 1984].
  - Need to be simple and effective in discriminate correctly between good and bad choices.
- A **heuristic search** is a search algorithm that uses information about
  - the initial state,
  - operators on finding the states adjacent to a state,
  - a test function whether a goal is reached, and
  - heuristics to pick the next state to explore.
- A "good" heuristic search algorithm:
  - States that are not likely leading to the goals will not be explored further.
    - ▷ *A state is cut or pruned.*
  - States are explored in an order that are according to their likelihood of leading to the goals → **good move ordering**.

Heuristic search: A*

- **Combining DFID with best-first heuristic search such as A*.**
- **A* search: branch and bound with a lower-bound estimation.**
- **Algorithm A$^*(N_0)$**
  - **Initialize a Priority Queue PQ to store partial paths with the key the cost of this path.**
    - ▷ ***Initially, store only a path with the starting node N_0 only.***
    - ▷ ***Paths in PQ are sorted according to their current cost plus a lower bound on the remaining distances.***
  - **While PQ is not empty do**
    - ▷ ***Remove a path P with the least cost from PQ***
    - ▷ ***11: If the goal is found, then return success***
    - ▷ ***12: Find extended paths from P by extending one step***
    - ▷ ***Insert all generated paths to PQ***
    - ▷ ***Update PQ***
    - ▷ ***If two paths reach a common node then keep only one with the least cost***
  - **Return fail**

A^* algorithm

- **Cost function:**
  - **Given a path P,**
    - ▷ ***let $g(P)$ be the current cost of P;***
    - ▷ ***let $h(P)$ be the estimation of remaining, or heuristic cost of P;***
    - ▷ ***$f(P) = g(P) + h(P)$ is the cost function.***
  - **How to find a good $h()$ is the key of an A^* algorithm?**
  - **It is known that if $h()$ never overestimates the actual cost to the goal (this is called admissible), then A^* always finds an optimal solution.**
    - ▷ ***Q: How to prove this?***
- **Checking of the termination condition:**
  - **We need check for a goal only when a path is popped from the PQ, i.e., at Line 11.**
  - **We cannot check for a goal when a path is generated and inserted into the PQ, i.e., at Line 12.**
    - ▷ ***We will not be able find the optimal solution if we do the checking at Line 12.***

A^* algorithm: Comments

- **When a path is inserted, check for whether it has reached some nodes that have been visited before.**
  - ▷ *It may take a huge space and a clever algorithm to implement an efficient Priority Queue.*
  - ▷ *It may need a clever data structure to efficiently check for possible duplications.*
- **Cost function:**
  - **Need an lower bound estimation that is as large as possible.**
  - **Can design the cost function so that A$*$ emulates the behavior of other search routines.**
- **It consumes a lot of memory to record the set of visited nodes.**
- **It also consume a lot of memory to store PQ.**
- **Q:**
  - ▷ *What disk based techniques can be used?*
  - ▷ *Why do we need a non-trivial $h(P)$ that is admissible?*
  - ▷ *How to design an admissible cost function?*

DFS with a threshold

- **DFS$_{cost}$($N, f, threshold$) is a version of DFS with a starting state N and a cost function f that cuts off a path when its cost is more than a given $threshold$.**
  - **DFS$_{depth}$(N, cut_off_depth) is a special version of DFS$_{cost}$($N, f, threshold$).**
- **Algorithm DFS$_{cost}$(N_0,f,$threshold$)**
  - **Initialize a Stack S**
  - **Push $(null, N_0)$ to S where N_0 is the initial state**
  - **While S is not empty do**
    - ▷ ***Pop*** $(current, N)$ ***from*** S
    - ▷ ***If*** $current$ ***is a goal, then return success*** **{∗ *Goal is found!* ∗}**
    - ▷ $R \leftarrow next(current, N)$ **{∗ *pick a good move ordering here* ∗}**
    - ▷ ***If*** $R = null$, ***then continue;*** **{∗ *searched all children of* N ∗}**
    - ▷ ***If*** $f(P) > threshold$, ***then continue where*** P ***is the current path*** **{∗ *cut off* ∗}**
    - ▷ ***Push*** (R, N) ***to*** S
    - ▷ ***If*** R ***is already in*** S, ***then continue*** **{∗ *to avoid loops* ∗}**
    - ▷ ***Push*** $(null, R)$ ***to*** S **{∗ *search deeper* ∗}**
  - **Return fail**

How to pick a good move ordering (1/2)

- **Instead of just using $next(current, N)$ to find the next unvisited neighbors of N with the information of the last visited node being $current$, we do the followings.**
  - **Use a routine to order the neighbors of N so that it is always the case the neighbors are visited from low cost to high cost.**
  - **Let this routine be $next1(current, N)$.**
  - **Note we still need dummy first and last elements being $null$.**

How to pick a good move ordering (2/2)

- **Algorithm DFS1$_{cost}$(N_0,f,$threshold$)**
  - **Initialize a Stack S**
  - **Push $(null, N_0)$ to S where N_0 is the initial state**
  - **While S is not empty do**
    - ▷ ***Pop*** $(current, N)$ ***from*** S
    - ▷ ***If*** $current$ ***is a goal, then return success***
    - ▷ $R \leftarrow next1(current, N)$
    - ▷ ***If*** $R = null$, ***then continue;*** **{*** ***searched all children of*** N ***}**
    - ▷ ***Let*** P ***be the path from*** N_0 ***to*** R
    - ▷ ***If*** $f(P) > threshold$, ***then continue*** **{*** ***cut off*** ***}**
    - ▷ ***Push*** (R, N) ***to*** S
    - ▷ ***If*** R ***is already in*** S, ***then continue*** **{*** ***to avoid loops*** ***}**
    - ▷ ***Push*** $(null, R)$ ***to*** S **{*** ***search deeper*** ***}**
  - **Return fail**

How to incooperate ideas from A*

- **Instead of using a stack in DFS$_{cost}$, use a priority queue.**
- **Algorithm DFS2$_{cost}$(N_0,f,$threshold$)**
  - **Initialize a priority queue PQ**
  - **Insert $(null, N_0)$ to PQ where N_0 is the initial state**
  - **While PQ is not empty do**
    - ▷ ***Remove $(current, N)$ with the least cost $f(P)$ for the path P from N_0 to N from PQ***
    - ▷ ***If $current$ is a goal, then return success***
    - ▷ $R \leftarrow next1(current, N)$
    - ▷ ***If $R = null$, then continue; {$*$ searched all children of N $*$}***
    - ▷ ***Let P be the path from N_0 to R***
    - ▷ ***If $f(P) > threshold$, then continue {$*$ cut off $*$}***
    - ▷ ***Insert (R, N) to PQ***
    - ▷ ***If R is already in PQ, then continue {$*$ to avoid loops $*$}***
    - ▷ ***Insert $(null, R)$ to PQ {$*$ search deeper $*$}***
  - **Return fail**
- **It may be costly to maintain a priority queue as in the case of A^*.**

IDA* = DFID + A*

- **DFS$_{cost}$($N, f, threshold$) is a version of DFS with a starting state N and a cost function f that cuts off a path when its cost is more than a given $threshold$.**
- **IDA*: iterative-deepening A***
- **Algorithm IDA*($N_0, threshold$)**
  - $threshold \leftarrow h(null)$
  - **While $threshold$ is reasonable do**
    - ▷ ***DFS$_{cost}$($N_0, g + h(), threshold$)***
      {$*$ *Can use DFS1$_{cost}$ or DFS2$_{cost}$ here* $*$}
    - ▷ ***If the goal is found,***
      then return success
    - ▷ $threshold \leftarrow$ ***the least*** $g(P) + h(P)$ ***cost among all paths*** P ***being cut***
  - **Return fail**

IDA*: comments

- IDA* does not need to use a priority queue as in the case of A*.
  - IDA* is optimal in terms of solution cost, time, and space over the class of admissible best-first searches on a tree.
- Issues in updating $threshold$.
  - Increase too little: re-search too often.
  - Increase too large: cut off too little.
  - Q: How to guarantee optimal solutions are not cut?
    - ▷ *It can be proved, as in the case of A*, that given an admissible cost function, IDA* will find an optimal solution, i.e., one with the least cost, if one exists.*
- Cost function is the knowledge used in searching.
- Combine knowledge and search!
- Need to balance the amount of time spent in realizing knowledge and the time used in searching.

15 puzzle (1/2)

- **Introduction of the game:**
  - 15 tiles in a 4*4 square with numbers from 1 to 15.
  - One empty cell.
  - A tile can be slided horizontally or vertically into an empty cell.
  - From an initial position, slide the tiles into a goal position.
- **Examples:**
  - Initial position:

10	8		12
3	7	6	2
1	14	4	11
15	13	9	5

  - Goal position:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

15 puzzle (2/2)

- **Total number of positions:** $16! = 20,922,789,888,000 \leq 2.1 * 10^{13}$.
  - **It is feasible, in terms of computation time, to enumerate all possible positions, since 2007.**
    - ▷ *Can use DFS or DFID now.*
    - ▷ *Need to avoid falling into loops or re-visit a node too many times.*
  - **It is still too large to store all possible positions in main memory now (2012).**
    - ▷ *Cannot use BFS efficiently even now.*
    - ▷ *Maybe difficult to find an optimal solution.*
    - ▷ *Maybe able to use disk based BFS.*

Solving 15 puzzles

- **Using DEC 2060 a 1-MIPS machine: solved the 15 puzzle problem within 30 CPU minutes for all testing positions, generating over 1.5 million nodes per minute.**
  - **The average solution length was 53 moves.**
  - **The maximum was 66 moves.**
  - **IDA* generated more nodes than A*, but ran faster due to less overhead per node.**
- **Heuristics used:**
  - $g(P)$**: the number of moves made so far.**
  - $h(P)$**: the Manhattan distance between the current board and the goal position.**
    - ▷ ***Suppose a tile is currently at*** (i, j) ***and its goal is at*** (i', j')***, then the Manhattan distance for this tile is*** $|i - i'| + |j - j'|$***.***
    - ▷ ***The Manhattan distance between a position and a goal position is the sum of the Manhattan distance of every tile.***
    - ▷ $h(P)$ ***is admissible.***

What else can be done?

- **Bi-directional search and IDA*?**
  - **How to design a good and non-trivial heuristic function?**
- **How to get a better move ordering in DFS?**
- **Balancing in resource allocation:**
  - **The efforts to memorize past results versus the amount of efforts to search again.**
  - **The efforts to compute a better heuristic, i.e., the cost function.**
  - **The amount of resources spent in implementing a better heuristic and the amount of resources spent in searching.**
- **Can these techniques be applied to two-person games?**

References and further readings

- Judea Pearl. Heuristics: Intelligent search strategies for computer problem solving. Assison-Wesley, 1984.

* R. E. Korf. Depth-first iterative-deepening: An optimal admissible tree search. *Artificial Intelligence*, 27:97–109, 1985.

- R. E. Korf and A. Felner. Disjoint pattern database heuristics. *Artificial Intelligence*, 134:9–22, 2002.