Scout and NegaScout

Tsan-sheng Hsu

徐讚昇

tshsu@iis.sinica.edu.tw

http://www.iis.sinica.edu.tw/~tshsu

Introduction

- It looks like alpha-beta pruning is the best we can do for a generic searching procedure.
  - What else can be done generically?
  - Alpha-beta pruning follows basically the "intelligent" searching behaviors used by human when domain knowledge is not involved.
  - Can we find some other "intelligent" behaviors used by human during searching?
- Intuition: MAX node.
  - Suppose we know currently we have a way to gain at least 300 points at the first branch.
  - If there is an efficient way to know the second branch is at most gaining 300 points, then there is no need to search the second branch in detail.
    - ▷ *Is there a way to search a tree approximately?*
    - ▷ *Is searching approximately faster than searching exactly?*
- Similar intuition holds for a MIN node.

SCOUT procedure

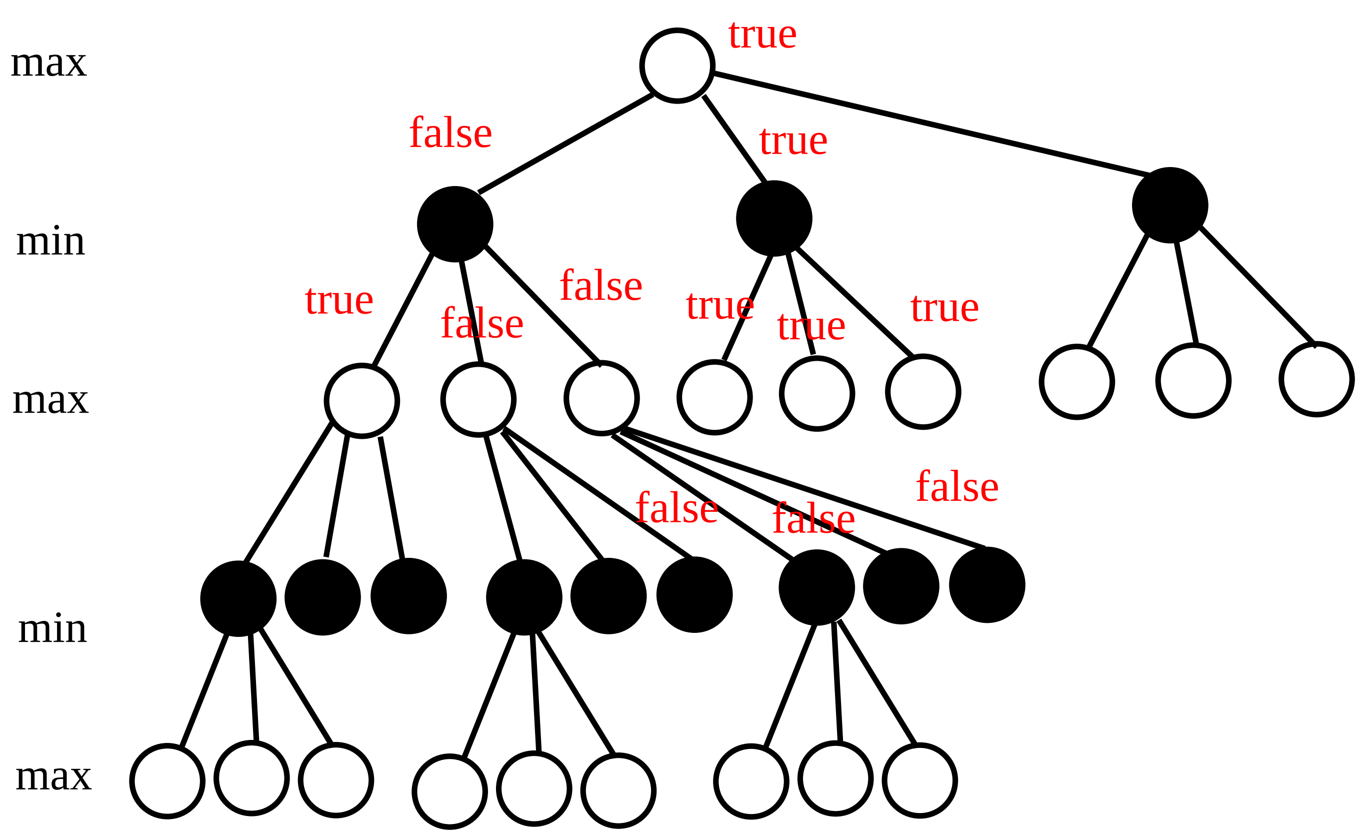
- **Invented by Judea Pearl in 1980.**
- **It may be possible to verify whether the value of a branch is greater than a value v or not in a way that is faster than knowing its exact value.**
- **High level idea:**
  - **While searching a branch T_b of a MAX node, if we have already obtained a lower bound v_ℓ.**
    - ▷ ***First TEST whether it is possible for T_b to return something greater than v_ℓ.***
    - ▷ ***If FALSE, then there is no need to search T_b.*** ***This is called fails the test.***
    - ▷ ***If TRUE, then search T_b.*** ***This is called passes the test.***
  - **While searching a branch T_c of a MIN node, if we have already obtained an upper bound v_u**
    - ▷ ***First TEST whether it is possible for T_c to return something smaller than v_u.***
    - ▷ ***If FALSE, then there is no need to search T_c.*** ***This is called fails the test.***
    - ▷ ***If TRUE, then search T_c.*** ***This is called passes the test.***

How to TEST $> v$

```
procedure TEST(position $p$, value $v$, condition $>$)
// test whether the value of the branch at $p$ is $> v$

- determine the successor positions $p_1, \ldots, p_d$ of $p$
- if $d = 0$, then // terminal
    ▷ if $f(p) > v$ then // f(): evaluating function
    ▷       return TRUE
    ▷ else return FALSE
- if $p$ is a MAX node, then
    • for $i := 1$ to $d$ do
        ▷ if TEST($p_i$, $v$, $>$) is TRUE, then
              return TRUE // succeed if a branch is $> v$
    • return FALSE // fail only if all branches $\leq v$
- if $p$ is a MIN node, then
    • for $i := 1$ to $d$ do
        ▷ if TEST($p_i$, $v$, $>$) is FALSE, then
              return FALSE // fail if a branch is $\leq v$
    • return TRUE // succeed only if all branches are $> v$
```

Illustration of TEST



How to TEST — Discussions

- **Condition can be stated as $<$ by properly revising the algorithm.**
  - $\boxed{\textbf{TEST}(p,v,>) \textbf{ is TRUE}} \equiv \boxed{\textbf{TEST}(p,v,<=) \textbf{ is FALSE}}$
  - $\boxed{\textbf{TEST}(p,v,>) \textbf{ is FALSE}} \equiv \boxed{\textbf{TEST}(p,v,<=) \textbf{ is TRUE}}$
  - $\boxed{\textbf{TEST}(p,v,<) \textbf{ is TRUE}} \equiv \boxed{\textbf{TEST}(p,v,>=) \textbf{ is FALSE}}$
  - $\boxed{\textbf{TEST}(p,v,<) \textbf{ is FALSE}} \equiv \boxed{\textbf{TEST}(p,v,>=) \textbf{ is TRUE}}$
- **Practical consideration:**
  - **Set a depth limit and evaluate the position's value when the limit is reached.**

How to TEST $< v$

```
procedure TEST(position $p$, value $v$, condition $<$)
// test whether the value of the branch at $p$ is $< v$

■ determine the successor positions $p_1, \ldots, p_d$ of $p$
■ if $d = 0$, then // terminal
    ▷ if $f(p) < v$ then // f(): evaluating function
    ▷         return TRUE
    ▷ else return FALSE
■ if $p$ is a MAX node, then
    • for $i := 1$ to $d$ do
        ▷ if TEST($p_i$, $v$, $<$) is FALSE, then
              return FALSE // succeed if a branch is $\geq v$
    • return TRUE // succeed only if all branches $< v$
■ if $p$ is a MIN node, then
    • for $i := 1$ to $d$ do
        ▷ if TEST($p_i$, $v$, $<$) is TRUE, then
              return TRUE // succeed if a branch is $< v$
    • return FALSE // fail only if all branches are $\geq v$
```

Main SCOUT procedure (1/2)

Algorithm SCOUT(position p)

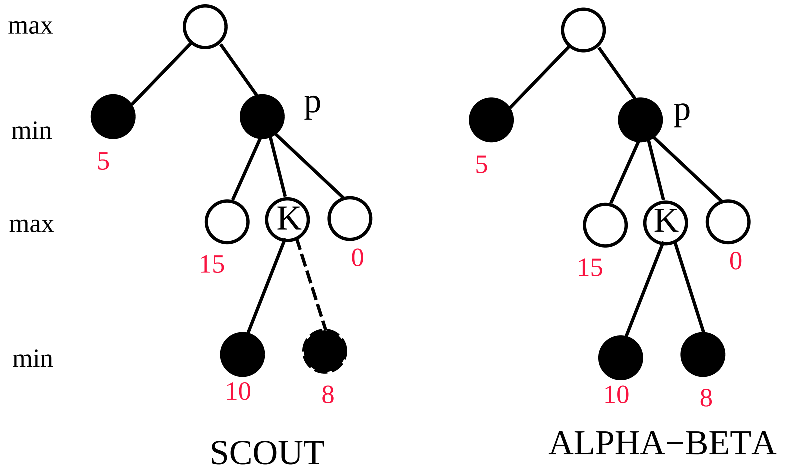
```
determine the successor positions $p_1, \ldots, p_d$
if $d = 0$, then return $f(p)$
             else $v = SCOUT(p_1)$ // SCOUT the first branch
if $p$ is a MAX node
    for $i := 2$ to $d$ do
        if TEST($p_i$, $v$, $>$) is TRUE, // TEST first for the rest of the branches
        then $v = SCOUT(p_i)$ // find the value of this branch if it can be $> v$
if $p$ is a MIN node
    for $i := 2$ to $d$ do
        if TEST($p_i$, $v$, $<$) is TRUE, // TEST first for the rest of the branches
        then $v = SCOUT(p_i)$ // find the value of this branch if it can be $< v$
return $v$
```

Main SCOUT procedure (2/2)

- **Note that v is the current best value at any moment.**
- **MAX node:**
  - **For any $i > 1$, if TEST(p_i, v, $>$) is TRUE,**
    - ▷ *then the value returned by $SCOUT(p_i)$ must be greater than v.*
  - **We say the p_i passes the test if TEST(p_i, v, $>$) is TRUE.**
- **MIN node:**
  - **For any $i > 1$, if TEST(p_i, v, $<$) is TRUE,**
    - ▷ *then the value returned by $SCOUT(p_i)$ must be smaller than v.*
  - **We say the p_i passes the test if TEST(p_i, v, $<$) is TRUE.**

Discussions for SCOUT (1/2)

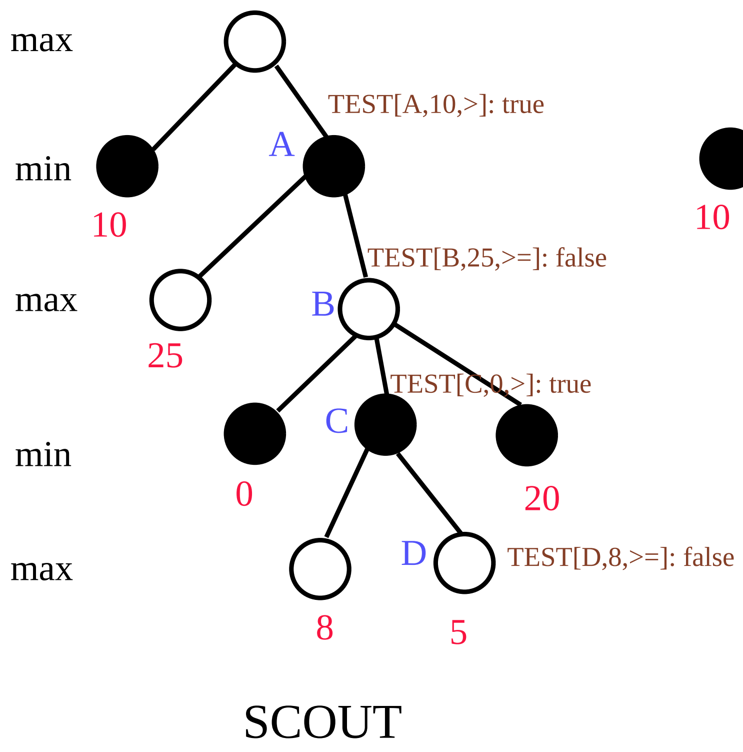
- **TEST who is called by SCOUT may visit less nodes than alpha-beta.**

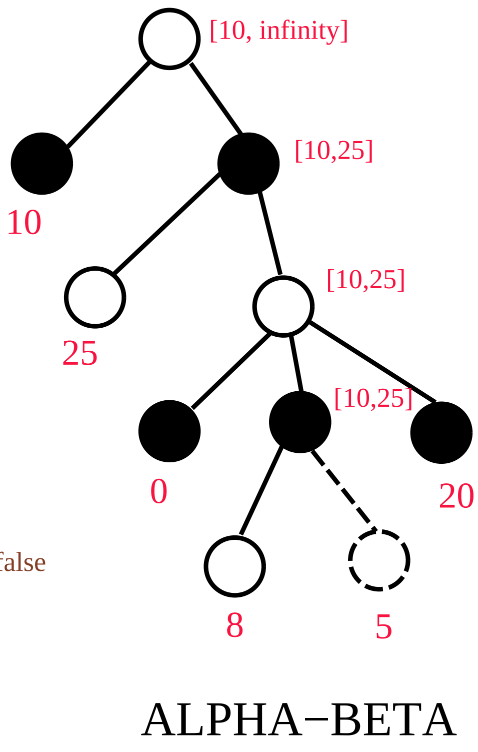


  - **Assume $TEST(p, 5, >)$ is called by the root after the first branch is evaluated.**
    - ▷ ***It calls $TEST(K, 5, >)$ which skips K's second branch.***
    - ▷ ***$TEST(p, 5, >)$ is FALSE, i.e., fails the test, after returning from the 3rd branch.***
    - ▷ ***No need to do SCOUT for the branch p.***
  - **Alpha-beta needs to visit K's second branch.**

Discussions for SCOUT (2/2)

- **SCOUT may pay many visits to a node that is cut off by alpha-beta.**





Number of nodes visited (1/3)

- **For TEST to return TRUE for a subtree T, it needs to evaluate at least**
  - ▷ ***one child for a MAX node in T, and***
  - ▷ ***and all of the children for a MIN node in T.***
  - ▷ ***If T has a fixed branching factor b and uniform depth d, the number of nodes evaluated is $\Omega(b^{d/2})$.***
- **For TEST to return FALSE for a subtree T, it needs to evaluate at least**
  - ▷ ***one child for a MIN node in T, and***
  - ▷ ***and all of the children for a MAX node in T.***
  - ▷ ***If T has a fixed branching factor b and uniform depth d, the number of nodes evaluated is $\Omega(b^{d/2})$.***

Number of nodes visited (2/3)

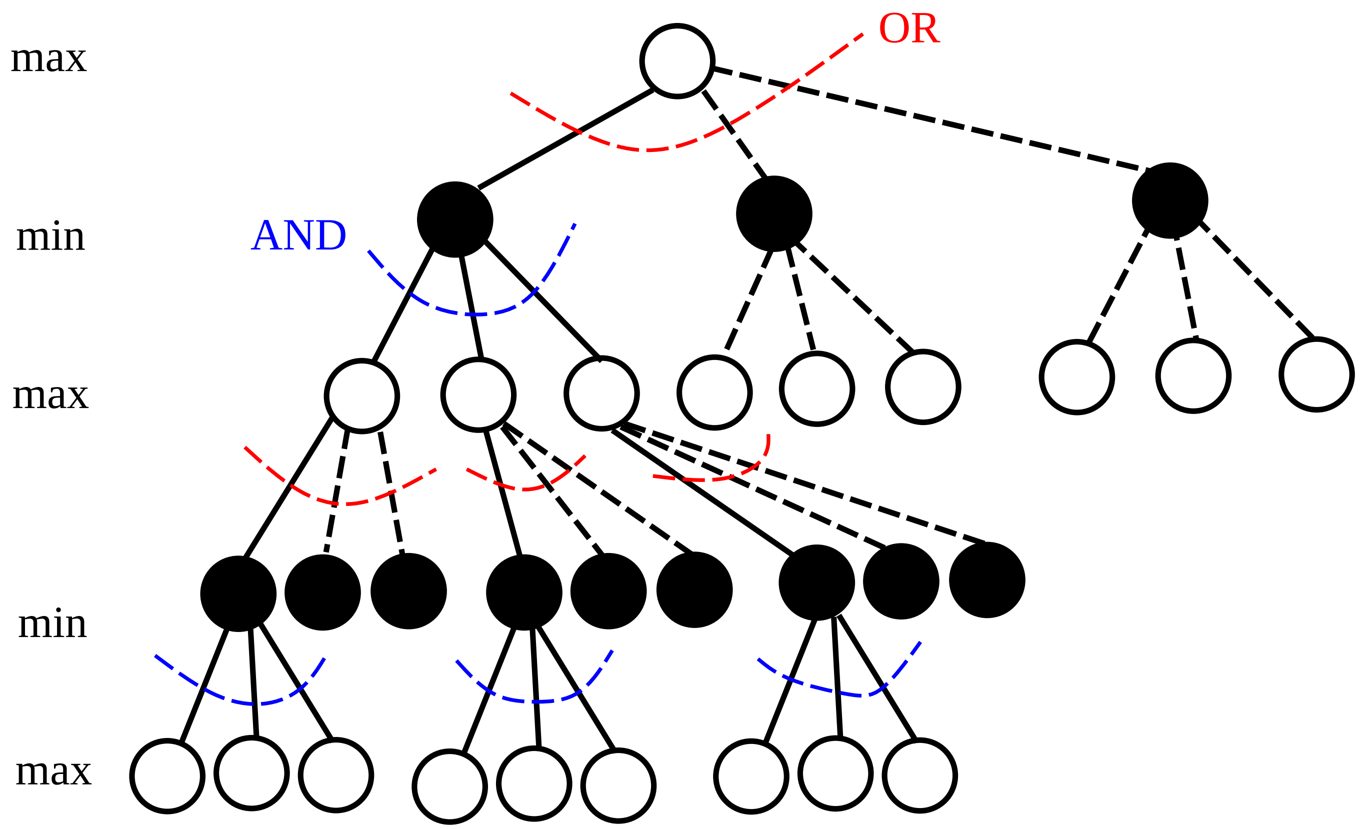
- **Assumptions:**
  - **Assume a full complete d-ary tree with depth ℓ.**
  - **Assume ℓ is even.**
  - **The depth of the root, which is a MAX node, is 0.**
- **The total number of nodes in the tree is $\frac{d^{\ell+1}-1}{d-1}$.**
- **The minimum number of nodes visited by TEST when it returns TRUE.**

$$\begin{aligned} &= 1+1+d+d+d^2+d^2+d^3+d^3+\cdots+d^{\ell/2-1}+d^{\ell/2-1}+d^{\ell/2} \\ &= 2\cdot(d^0+d^1+\cdots+d^{\ell/2})-d^{\ell/2} \\ &= 2\cdot\frac{d^{\ell/2+1}-1}{d-1}-d^{\ell/2} \end{aligned}$$

- **The minimum number of nodes visited by alpha-beta.**

$$\begin{aligned} &= \sum_{i=0}^{\ell} d^{\lceil i/2\rceil}+d^{\lfloor i/2\rfloor}-1 \\ &= 1+d+(2d-1)+(d^2+d-1)+\cdots+(d^{\ell/2}+d^{\ell/2-1}-1)+(2\cdot d^{\ell/2}-1) \end{aligned}$$

Number of nodes visited (3/3)



Comparisons

- **When the first branch of a node has the best value, then TEST scans the tree fast.**
  - **The best value of the first $i-1$ branches is used to test whether the ith branch needs to be searched exactly.**
  - **If the value of the first $i-1$ branches of the root is better than the value of ith branch, then we do not have to evaluate exactly for the ith branch.**
- **Compared to alpha-beta pruning whose cut off comes from bounds of search windows.**
  - **It is possible to have some cut-off for alpha-beta as long as there are some relative move orderings are "good."**
    - ▷ *The moving orders of your children and the children of your ancestor who is odd level up decide a cut-off.*
  - **The search bound is updated during the searching.**
    - ▷ *Sometimes, a deep alpha-beta cut-off occurs because a bound found from your ancestor a distance away.*

Performance of SCOUT (1/2)

- **A node may be visited more than once.**
  - **First visit is to TEST.**
  - **Second visit is to SCOUT.**
    - ▷ *During SCOUT, it may be TESTed with a different value.*
  - **Q: Can information obtained in the first search be used in the second search?**
- **SCOUT is a recursive procedure.**
  - **A node in a branch that is not the first child of a node with a depth of ℓ.**
    - ▷ *Note that the depth of the root is defined to be 0.*
    - ▷ *Every ancestor of you may initiate a TEST to visit you.*
    - ▷ *It can be visited ℓ times by TEST.*

Performance of SCOUT (2/2)

- **Show great improvements on $depth > 3$ for games with small branching factors.**
  - **It traverses most of the nodes without evaluating them preciously.**
  - **Few subtrees remained to be revisited to compute their exact mini-max values.**
- **Experimental data show [Pearl 1980]:**
  - **SCOUT favors “skinny” games, that are games with high depth-to-width ratios.**
  - **On depth = 5, it saves over 40% of time.**
  - **Maybe bad for games with a large branching factor.**
  - **Move ordering is very important.**
    - ▷ ***The first branch, if is good, offers a great chance of pruning further branches.***

Alpha-beta revisited

- **In an alpha-beta search with a window [$alpha$,$beta$]:**
  - **Failed-high means it returns a value that is larger than its upper bound $beta$.**
  - **Failed-low means it returns a value that is smaller than its lower bound $alpha$.**
- **Null or Zero window search:**
  - **Using alpha-beta search with the window [m,$m+1$].**
  - **The result can be either failed-high or failed-low.**
  - **Failed-high means the return value is at least $m+1$.**
    - ▷ ***Equivalent to $TEST(p, m, >)$ is true.***
  - **Failed-low means the return value is at most m.**
    - ▷ ***Equivalent to $TEST(p, m, >)$ is false.***

Alpha-Beta + Scout

- **Intuition:**
  - **Try to incooperate SCOUT and alpha-beta together.**
  - **The searching window of alpha-beta if properly set can be used as TEST in SCOUT.**
  - **Using a searching window is better than using a single bound as in SCOUT.**
  - **Can also apply alpha-beta cut if it applies.**
- **Modifications to the SCOUT algorithm:**
  - **Traverse the tree with two bounds as the alpha-beta procedure does.**
    - ▷ ***A searching window.***
    - ▷ ***Use the current best bound to guide the TEST value.***
  - **Use a fail soft version to get a better result when the returned value is out of the window.**

The NegaScout Algorithm – MiniMax (1/2)

- **Algorithm $F4'$(position p, value $alpha$, value $beta$, integer $depth$)**

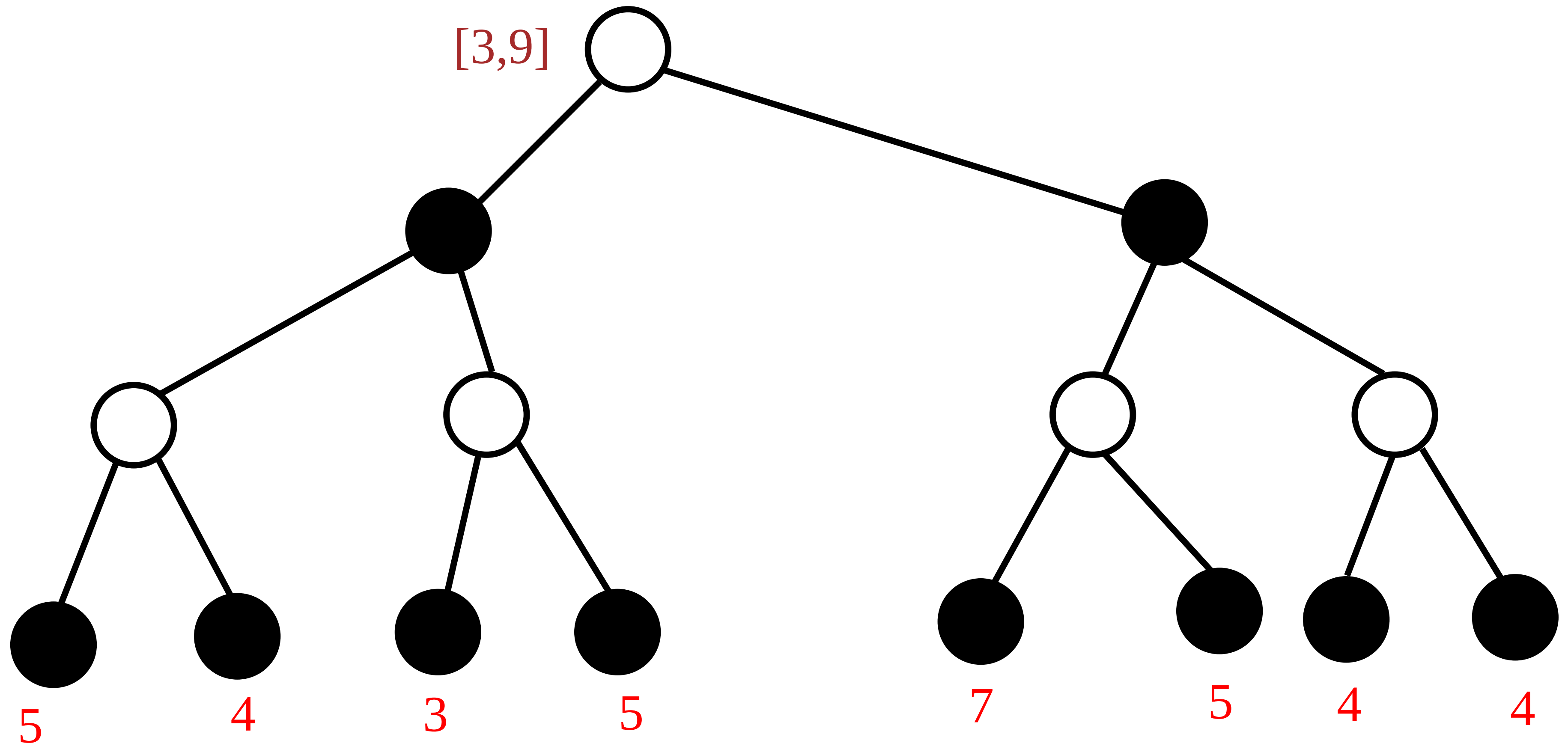
```
• determine the successor positions $p_1, \ldots, p_d$
• if $d = 0$ // a terminal node
    or $depth = 0$ // $depth$ is the remaining depth to search
    or time is running up // from timing control
    or some other constraints are met // apply heuristic here
• then return $f(p)$ else
  begin
      ▷ $m := -\infty$ // $m$ is the current best lower bound; fail soft
        $m := \max\{m, G4'(p_1, alpha, beta, depth - 1)\}$ // the first branch
        if $m \geq beta$ then return($m$) // beta cut off
      ▷ for $i := 2$ to $d$ do
      ▷ 9:    $t := G4'(p_i, m, m + 1, depth - 1)$ // null window search
      ▷ 10:   if $t > m$ then // failed-high
        11:       if ($depth < 3$ or $t \geq beta$)
        12:       then $m := t$
        13:       else $m := G4'(p_i, t, beta, depth - 1)$ // re-search
      ▷ 14:   if $m \geq beta$ then return($m$) // beta cut off
  end
• return $m$
```

The NegaScout Algorithm – MiniMax (2/2)

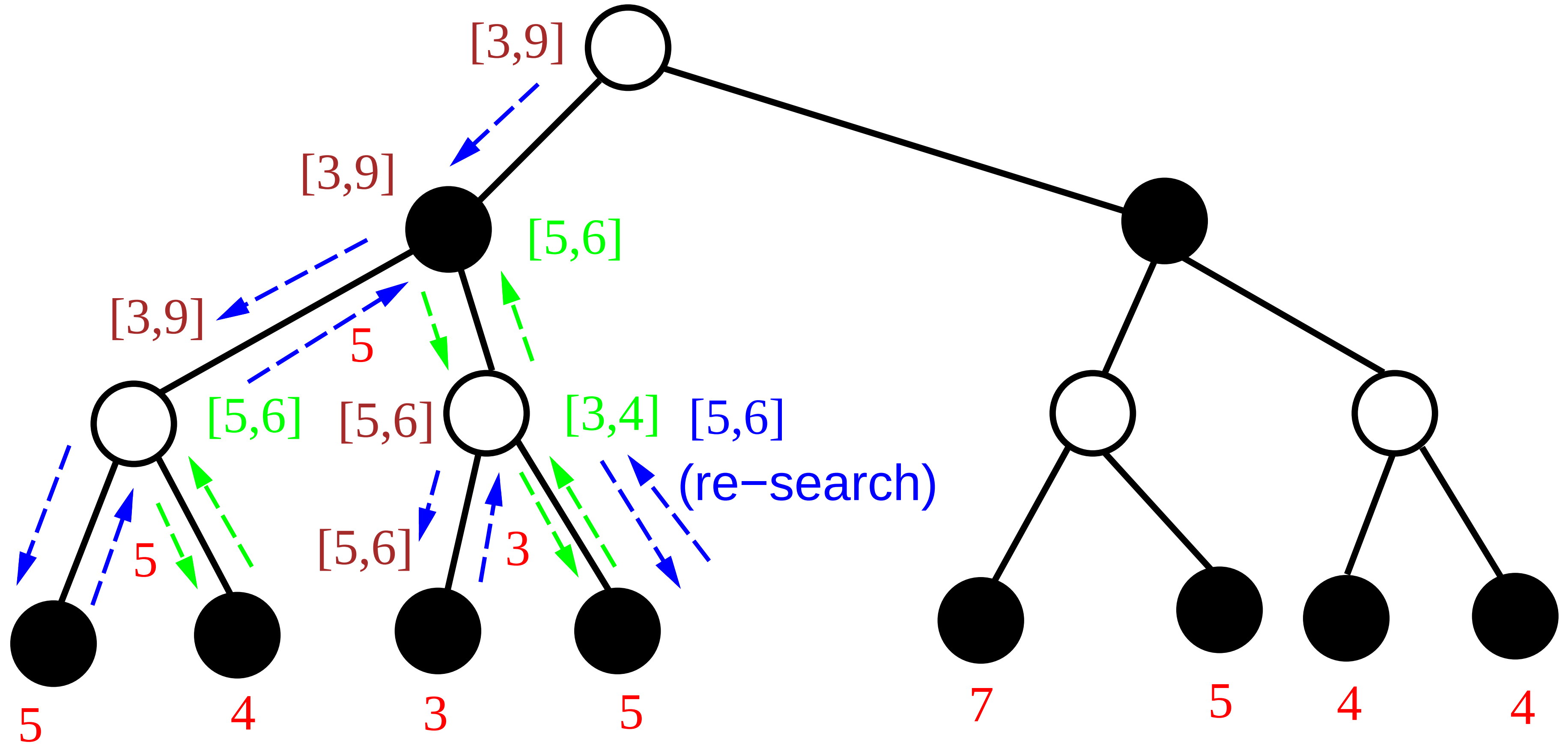
- **Algorithm $G4'$(position p, value $alpha$, value $beta$, integer $depth$)**

```
• determine the successor positions $p_1, \ldots, p_d$
• if $d = 0$ // a terminal node
    or $depth = 0$ // $depth$ is the remaining depth to search
    or time is running up // from timing control
    or some other constraints are met // apply heuristic here
• then return $f(p)$ else
  begin
      ▷ $m = \infty$ // $m$ is the current best upper bound; fail soft
        $m := \min\{m, F4'(p_1, alpha, beta, depth - 1)\}$ // the first branch
        if $m \leq alpha$ then return($m$) // alpha cut off
      ▷ for $i := 2$ to $d$ do
      ▷ 9:      $t := F4'(p_i, m, m+1, depth - 1)$ // null window search
      ▷ 10:     if $t <= m$ then // failed-low
        11:         if ($depth < 3$ or $t \leq alpha$)
        12:         then $m := t$
        13:         else $m := F4'(p_i, alpha, t, depth - 1)$ // re-search
      ▷ 14:     if $m \leq alpha$ then return($m$) // alpha cut off
  end
• return $m$
```

NegaScout – MiniMax version (1/2)



NegaScout – MiniMax version (2/2)



The NegaScout Algorithm

- Use Nega-MAX format.
- Algorithm $F4$(position p, value $alpha$, value $beta$, integer $depth$)

```
• determine the successor positions $p_1, \ldots, p_d$
• if $d = 0$ // a terminal node
    or $depth = 0$ //$depth$ is the remaining depth to search
    or time is running up // from timing control
    or some other constraints are met // apply heuristic here
• then return $h(p)$ else
    ▷ $m := -\infty$ // the current lower bound; fail soft
    ▷ $n := beta$ // the current upper bound
    ▷ for $i := 1$ to $d$ do
    ▷  9:   $t := -F4(p_i, -n, -max\{alpha, m\}, depth - 1)$
    ▷ 10:   if $t > m$ then
      11:        if ($n = beta$ or $depth < 3$ or $t \geq beta$)
      12:        then $m := t$
      13:        else $m := -F4(p_i, -beta, -t, depth - 1)$ // re-search
    ▷ 14:   if $m \geq beta$ then return($m$) // cut off
    ▷ 15:   $n := max\{alpha, m\} + 1$ // set up a null window
• return $m$
```

Search behaviors (1/3)

- **If the depth is enough or it is a terminal position, then stop searching further.**
  - **Return $h(p)$ as the value computed by an evaluation function.**
  - **Note:**

$$h(p) = \begin{cases} f(p) & \text{if depth of } p \text{ is 0 or even} \\ -f(p) & \text{if depth of } p \text{ is odd} \end{cases}$$

- **Fail soft version.**
- **For the first child p_1, search using the normal alpha beta window..**
  - **line 9: normal window for the first child**
  - **the initial value of m is $-\infty$, hence $-max\{alpha, m\} = -alpha$**
    - ▷ *m is current best value*
  - **that is, searching with the normal window $[alpha, beta]$**

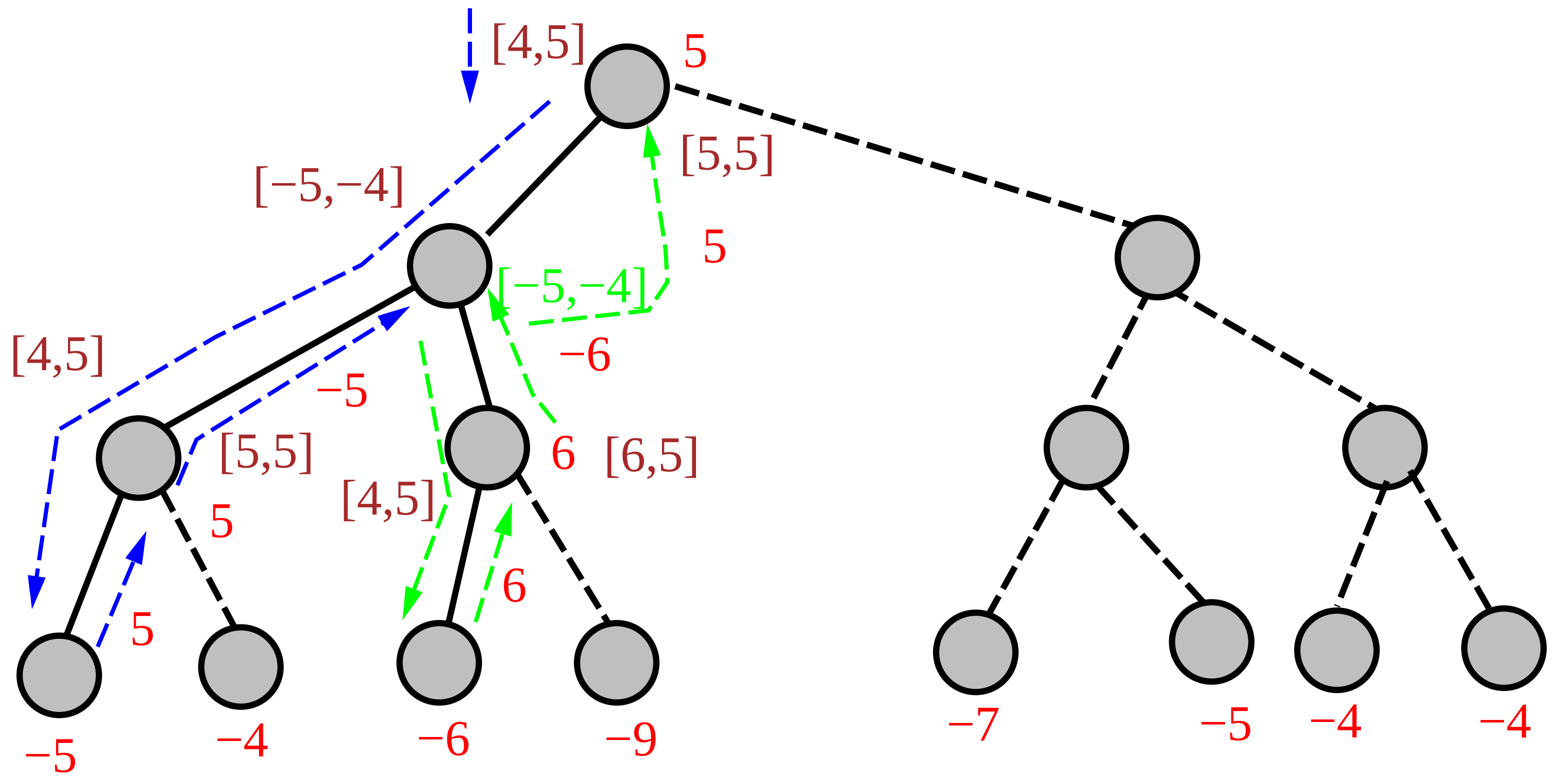
Search behaviors (2/3)

- **For the second child and beyond p_i, $i > 1$, first perform a null window search for testing whether m is the answer.**
  - **line 9: a null-window of $[m, m + 1]$ searches for the second child and beyond.**
    - ▷ ***m is best value obtained so far***
    - ▷ ***m's value will be first set at line 12 because $n = beta$***
    - ▷ ***The null window is set at line 15.***
  - **line 11:**
    - ▷ ***$n = beta$: we are at first iteration.***
    - ▷ ***$depth < 3$: on a smaller depth subtree, i.e., depth at most 2, NegaScout always returns the best value.***
    - ▷ ***$t \geq beta$: we have obtained a good enough value from the failed-soft version to guarantee a beta cut.***

Search behaviors (3/3)

- **For the second child and beyond p_i, $i > 1$, first perform a null window search for testing whether m is the answer.**
  - **line 11: on a smaller depth subtree, i.e., depth at most 2, NegaScout always returns the best value.**
    - ▷ *Normally, no need to do alpha-beta or any enhancement on very small subtrees.*
    - ▷ *The overhead is too large on small subtrees.*
  - **line 13: re-search when the null window search fails high.**
    - ▷ *The value of this subtree is at least t.*
    - ▷ *This means the best value in this subtree is more than m, the current best value.*
    - ▷ *This subtree must be re-searched with the the window $[t, beta]$.*
  - **line 14: the normal pruning from alpha-beta.**

Example for NegaScout



Refinements

- **When a subtree is re-searched, it is best to use information on the previous search to speed up the current search.**
    - Restart from the position that the value t is returned.
- **Maybe want to re-search using the normal alpha-beta procedure.**
- **$F4$ runs much better with a good move ordering and transposition tables.**
    - Order the moves in a best-first list.
    - Reduce the number of re-searches.

Performances

- **Experiments done on a uniform random game tree [Reinefeld 1983].**
  - **Normally superior to alpha-beta when searching game tree with branching factors from 20 to 60.**
  - **Shows about 10 to 20% of improvement.**

Comments

- Incooperating both SCOUT and alpha-beta.
- Used in state-of-the-art game search engines.
- The first search, though maybe unsuccessful, can provide useful information in the second search.
    - Information can be stored and then be reused.

References and further readings

* A. Reinefeld. An improvement of the scout tree search algorithm. *ICCA Journal*, 6(4):4–14, 1983.
* J. Pearl. Asymptotic properties of minimax trees and game-searching procedures. *Artificial Intelligence*, 14(2):113–138, 1980.