Monte-Carlo Game Tree Search: Advanced Techniques

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Abstract

- **Adding new ideas to the pure Monte-Carlo approach for computer Go.**
  - **On-line knowledge: domain independent techniques**
    - ▷ *Progressive pruning (PP)*
    - ▷ *All moves as first (AMAF) and RAVE heuristic*
    - ▷ *Node expansion policy*
    - ▷ *Temperature*
    - ▷ *Depth-i tree search*
  - **Machine learning and deep learning: domain dependent techniques**
    - ▷ *Node expansion*
    - ▷ *Better simulation policy*
    - ▷ *Better position evaluation*
- **Conclusion:**
  - **Combining the power of statistical tools and machine learning, Monte-Carlo approach reaches a new high for computer Go.**

Domain independent refinements

- **Main considerations**
  - **Avoid doing un-needed computations**
  - **Increase the speed of convergence**
  - **Avoid early mis-judgement**
  - **Avoid extreme cases**
- **Refinements came from on-line knowledge.**
  - **Progressive pruning.**
    - ▷ *Cut hopeless nodes early.*
  - **All moves at first and RAVE.**
    - ▷ *Increase the speed of convergence.*
  - **Node expansion policy.**
    - ▷ *Grow only nodes with a potential.*
  - **Temperature.**
    - ▷ *Introduce randomness.*
  - **Depth-i enhancement.**
    - ▷ *With regard the initial phase, the one on obtaining an initial game tree, exhaustively enumerate all possibilities instead of using only the root.*

Warning

- Many of the domain independent refinements are invented earlier than the idea of UCT tree search.
- For a better flow of introduction, UCT is introduced earlier.
- These domain independent techniques can be used with or without UCT.
- These techniques speed up the convergence rate, but cannot really replace the importance of getting more simulations.
  - If the amount of simulations performed is well enough, then you can most likely find a good answer without using those techniques. In the worst case, you will be hurt by spending more time to do these additional techniques.
  - In the extreme case, if you can do well enough simulations, then no UCB formula is needed at all.
- Lesson: Do enough, but not over, simulations for the problem instance under the current resource constraint.

Progressive pruning (1/5)

- **Each position has a mean value μ and a standard deviation σ after performing some simulations.**
  - **Left expected outcome $\mu_l = \mu - r_d * \sigma$.**
  - **Right expected outcome $\mu_r = \mu + r_d * \sigma$.**
  - **The value r_d is a constant fixed up empirically.**
- **Let P_1 and P_2 be two child positions of a position P.**
- **P_1 is *statistically inferior* to P_2 if $P_1.\mu_r < P_2.\mu_l$, and $P_1.\sigma < \sigma_e$ and $P_2.\sigma < \sigma_e$.**
  - **The value σ_e is called *standard deviation for equality*.**
  - **Its value is determined empirically.**
- **P_1 and P_2 are *statistically equal* if $P_1.\sigma < \sigma_e$, $P_2.\sigma < \sigma_e$ and no move is statistically inferior to the other.**

Progressive pruning (2/5)

- **After a minimal number of random games, say 100 per move, a position is pruned as soon as it is statistically inferior to another.**
  - **For a pruned position:**
    - ▷ *Not considered as a legal move.*
    - ▷ *No need to maintain its UCB information.*
  - **This process is stopped when**
    - ▷ *this is the only one move left for its parent, or*
    - ▷ *the moves left are statistically equal, or*
    - ▷ *a maximal threshold, say 10,000 multiplied by the number of legal moves, of iterations is reached.*
- **Two different pruning rules.**
  - **Hard: a pruned move cannot be a candidate later on.**
  - **Soft: a move pruned at a given time can be a candidate later on if its value is no longer statistically inferior to a currently active move.**
    - ▷ *The score of an active move may be decreased when more simulations are performed.*
    - ▷ *Periodically check whether to reactive it.*

Progressive pruning (3/5)

- **Remarks:**
  - **Assume each trial is an independent Bernoulli trial and hence the distribution is normal.**
    - ▷ *This needs to be checked in your application.*
  - **We only compare nodes that are of the same parent.**
  - **We usually compare their raw scores not their UCB values.**
    - ▷ *UCB and PP are similar in ideas, but using different pre-assumptions.*
  - **If you compare UCB scores, then the mean and standard deviation of a move are those calculated only from its un-pruned children.**
- **Experimental setup:**
  - **9 by 9 Go.**
  - **Difference of stones plus eyes after Komi is applied.**
  - **The experiment is terminated if any one of the followings is true.**
    - ▷ *There is only move left for the root.*
    - ▷ *All moves left for the root are statistically equal.*
    - ▷ *A given number of simulations are performed.*
  - **The baselines of the experiments are those with scores 0.**

Progressive pruning (4/5)

- **Selection of r_d.**
  - **The greater r_d is,**
    - ▷ *the less pruned the moves are;*
    - ▷ *the better the algorithm performs;*
    - ▷ *the slower the play is.*
  - **Results [Bouzy et al'04]:**

r_d	1	2	4	8
score	0	+ 5.6	+ 7.3	+9.0
time	10'	35'	90'	150'

- **Selection of σ_e.**
  - **The smaller σ_e is,**
    - ▷ *the fewer equalities there are;*
    - ▷ *the better the algorithm performs;*
    - ▷ *the slower the play is.*
  - **Results [Bouzy et al'04]:**

σ_e	0.2	0.5	1
score	0	-0.7	-6.7
time	10'	9'	7'

- **Conclusions:**
  - **r_d plays an important role in the move pruning process.**
  - **σ_e is less sensitive.**

Progressive pruning (5/5)

- **Comments:**
  - **It makes little sense to compare nodes of different depths or belonging to different players.**
  - **Another trick that may need consideration is progressive widening or progressive un-pruning.**
    - ▷ *A node is effective if enough simulations are done on it and its values are good.*
  - **Note that we can set a threshold on whether to expand or grow the end of the selected $\mathbf{PV}_{UCB}$ path.**
    - ▷ *This threshold can be enough simulations are done and/or the score is good enough.*
    - ▷ *Use this threshold to control the way the underline tree is expanded.*
    - ▷ *If this threshold is high, then it will not expand any node and looks like the original version.*
    - ▷ *If this threshold is low, then we may make not enough simulations for each node in the underline tree.*
  - **If you want to do the above, you need to use a hash table to store the number of simulations done a node and its win rate.**

Comments in using PP

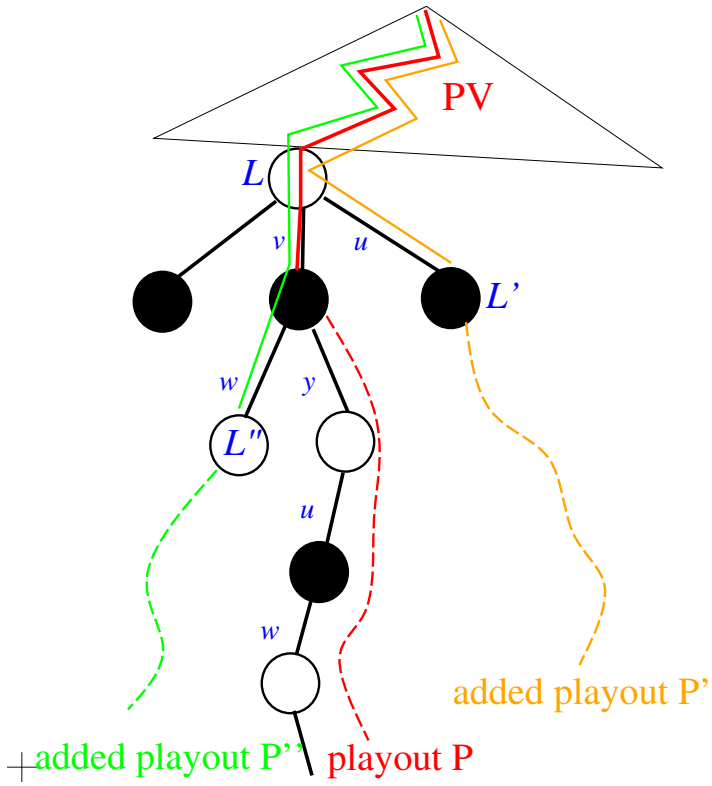
- **Important remarks:**
  - **Ideas for using the confidence interval on PP and the ideas for using upper and lower confidence bounds (LCB and UCB) are similar.**
  - **For PP to work, it is better to work on a version of UCB scoring formula not only on win rates.**
    - ▷ *If the result of a simulation can only be 0 or 1, then the mean of a sampling uniquely determines its standard deviation.*
    - ▷ *If the result of a simulation can have only very few variations, e.g., -1, 0, 1, then there are only a few possible standard deviations once the mean of a sampling is given.*
  - **The range of possible scores is important in using PP.**
    - ▷ *A very narrow range makes the cutting not very flexible.*
    - ▷ *A very wide range makes the cutting too random.*

All-moves-as-first heuristic (AMAF)

- **How to perform statistics for a completed playout?**
  - **Basic idea: its score is used for the first move of the game only.**
  - **All-moves-as-first AMAF: its score is used for all moves played in the game as if they were the first to be played [Bruegmann'93].**
- **AMAF updating rules:**
  - **If a playout S, starting from the position following $\mathbf{PV}_{UCB}$ towards the best leaf and then appending a simulation run, passes through a position V from W with a sibling position U, then**
    - ▷ *the counters at the position V leads to is updated;*
    - ▷ *the counters at the node U leads to is also updated if S later contains a ply from W to U.*
  - **Note, we apply this update rule for all nodes in S regardless nodes made by the player that is different from the root player.**

Illustration: AMAF

- **Assume a playout P is simulated from the root with the sequence of plys starting from the position L being v, y, u, w, $\cdots$.**
- **The winning rates of nodes along this path are updated.**
- **The winning rates of node L', a child of L, and node L'', a descendent of L, are also updated.**
  - ▷ ***In the added playout P' at L', exchange u and v in the playout.***
  - ▷ ***In the added playout P'' at L'', exchange w and y in the playout.***
- **In this example, 3 playouts are recorded for the position L though only one is performed.**
- **Note: Need to also update the exploration scores of affected nodes.**



AMAF: Implementation

- **When a playout, say $P_1, P_2, \ldots, P_h$ is simulated where P_1 is the root position of the selected $\mathbf{PV}_{UCB}$ and P_h is the end position of the playout, then we perform the following updating operations bottom up:**
  - $count := 1$
  - **for** $i := h - 1$ **downto** 1 **do**
    - ▷ *for each child position W of P_i that is not equal to P_{i+1} do*
    - ▷ *if the ply $(P_i \to W)$ is played in $P_i, P_{i+1}, \ldots, P_h$ then*
    - ▷ {
    - ▷ *update the score and counters of W;*
    - ▷ $count += 1$;
    - ▷ }
    - ▷ *update the score and counters of P_i as though $count$ playouts are performed*
- **Some forms of hashing is needed to check the if condition efficiently.**
- **It is better to use a good data structure to record the children of a position when it is first generated to avoid regenerating.**

AMAF: Pro's and Con's

- **Advantage:**
  - **All-moves-as-first helps speeding up the convergence of the simulations.**
- **Drawbacks:**
  - **The evaluation of a move from a random game in which it was played at a late stage is less reliable than when it is played at an early stage.**
  - **Recapturing.**
    - ▷ *Order of moves is important for certain games.*
    - ▷ *Modification: if several moves are played at the same place because of capturing, modify only the statistics for the player who played first.*
  - **Some move is good only for one player.**
    - ▷ *It does not evaluate the value of an intersection for the player to move, but rather the difference between the values of the intersections when it is played by one player or the other.*

AMAF: results

- **Results [Bouzy et al'04]:**
  - **Relative scores between different heuristics.**

AMAF	basic idea	PP
0	+13.7	+ 4.0

    - ▷ *Basic idea is very slow: 2 hours vs 5 minutes.*
  - **Number of random games N: relative scores with different values of N using AMAF.**

N	1000	10000	100000
score	-12.7	0	+3.2

    - ▷ *Using the value of 10000 is better.*
- **Comments:**
  - **The statistical natural is something very similar to the history heuristic as used in alpha-beta based searching.**

AMAF refinements

- **Definitions:**
  - **Let $v_1(P)$ be the score of a position P without using AMAF.**
  - **Let $v_2(P)$ be the score of a position P with AMAF.**
    - ***In calculating $v_2(P)$ we need to take into consideration all playouts, actual and added ones.***
    - ***It is odd to use only added playouts to compute.***
- **Remark: $v_2(P)$ uses both information of actual playouts and the added playouts from AMAF, while $v_1(P)$ uses only information from actual playouts only.**
- **Observations:**
  - **$v_1(P)$ is a good indicator for the goodness of P when sufficient number of trials are performed starting with P.**
  - **$v_2(P)$ is a good guess for the goodness of P for the true score of the position P when**
    - ***it is approaching the end of a game;***
    - ***too few trials are performed starting with P such as when the node for P is first expanded.***
- **Q: How to make the best use of $v_1(P)$ and $v_2(P)$ together?**

RAVE

- **Definitions:**
  - **Let $v_1(P)$ be the score of a position P without using AMAF.**
  - **Let $v_2(P)$ be the score of a position P with AMAF.**
- **Rapid Action Value Estimate (RAVE) [Silver'09]**
  - **Let the revised score $v_3(P) = \alpha \cdot v_1(P) + (1-\alpha) \cdot v_2(P)$ with a properly chosen value of α.**
    - ▷ ***Other formulas for mixing the two scores exist.***
  - **Can dynamically change α as the game goes.**
    - ▷ ***For example:*** $\alpha = \min\{1, \frac{N_P}{10000}\}$, ***where*** N_P ***is the number of playouts done on*** P.
    - ▷ ***This means when*** N_P ***reaches 10000, no AMAF is used.***
- $v_3(P) = \alpha \cdot v_1(P) + (1-\alpha) \cdot v_2(P)$
  - **When $\alpha = 0$, it is pure AMAF.**
  - **When $\alpha = 1$, it uses no AMAF.**

Other formulations of RAVE (1/2)

- **Note:** $v_3(P) = \alpha \cdot v_1(P) + (1 - \alpha) \cdot v_2(P)$
- **Example: Silver in his 2009 Ph.D. thesis [Silver'09] originally set the parameters as follows:**
  - **Let $\tilde{N}_P = N_P + N'_P$ where N_P is the number of actual simulations done at the position P and N'_P is the number of extra added simulations generated from AMAF at P.**
    - ***$\tilde{N}_P$ is the total number of simulations (actual and added) used to generate the AMAF score $v_2(P)$.***
    - ***N_P is the total number of actual simulations used to generate $v_1(P)$.***
  - **$1 - \alpha = \beta = \frac{\tilde{N}_P}{N_P + \tilde{N}_P + 4b^2 N_P \tilde{N}_P}$ where b is a constant to be decided empirically.**
  - **Namely, $v_3(P) = (1 - \beta) \cdot v_1(P) + \beta \cdot v_2(P)$**

Other formulations of RAVE (2/2)

- **Note:** $v_3(P) = (1-\beta) \cdot v_1(P) + \beta \cdot v_2(P)$
- **Discussion:**
  - $\beta = \frac{1}{\frac{N_P}{\tilde{N}_P} + 1 + 4b^2 N_P}$
  - **We know** $\tilde{N}_P \geq N_P$**, hence** $\frac{1}{2+4b^2 N_P} \leq \beta \leq \frac{1}{1+4b^2 N_P}$**.**
  - **When** $N_P >> 1/(4b^2)$ **is large, then** $\beta \to 0$ **which means uses mostly information in** $v_1(P)$**.**
    - ▷ ***When*** N_P ***is small,*** β ***is larger.***
  - **For the same** $\tilde{N}_P$**, if** N_P **is smaller, then** β **is larger, which means using more information in** $v_2(P)$**.**
- **Comments:**
  - **Silver is the first one to propose RAVE, but we choose to introduce a simpler formulation earlier for ease of description.**

Node expansion

- **May decide to expand potentially good nodes judging from the current statistics [Yajima et al'11].**
  - **All ends: expand all possible children of a newly added node.**
  - **Visit count: delay the expansion of a node until it is visited a certain number of times.**
  - **Transition probability: delay the expansion of a node until its "score" or estimated visit count is high comparing to that of its siblings.**
    - ▷ *Use the current mean, variance and parent's values to derive a good estimation using statistical methods.*
- **Expansion policy with some transition probability is much better than the "all ends" or pure "visit count" policy.**

Temperature (1/2)

- **Constant temperature: consider all the legal moves and play the ith move with a probability proportional to $e^{(K \cdot v_i)}$, where**
  - **v_i is the current value of the position obtained by taking move i;**
    - ***It is usually the case $v_i \geq 0$.***
    - ***$e^{(K \cdot v_i)} \geq 1$.***
  - **$K \geq 0$ is the inverse of the temperature T used in a simulated annealing setting.**
    - ***Add extra randomness by setting a constant $K = 1/T$.***
    - ***The probability of playing the ith move is $P_i(K) = \frac{e^{K \cdot v_i}}{\sum_{\forall q} e^{K \cdot v_q}}$.***
    - ***When $K \to 0$, which means temperature $T \to \infty$, and the selection is uniformly random.***
    - ***If $v_i > v_j$ and $K_1 > K_2$, then $P_i(K_1) - P_j(K_1) > P_i(K_2) - P_j(K_2)$.***
      $\to$ When K becomes larger,
      the value of v_i contributes more in the calculation of $P_i(K)$.
    - ***When K is very large, which means temperature is very low, it looks like some form of the "greedy", or best first, approach.***

Temperature (2/2)

- **Results for using a constant temperature [Bouzy et al'04]:**

K	0	2	5	10	20
score	-8.1	0	+2.6	-4.9	-11.3

  - When temperature is very high ($K = 0$) when means pure random, then it looks bad.
  - When there is no added randomness ($K > 5$), it also looks bad.
  - Tradeoff between the current score and randomness.
    - ▷ *Currently, a greedy approach is worse than a random approach!!!*

- **Simulated annealing (temperature decreasing, or K increasing):** $P_i(K_t) = \frac{e^{K_t \cdot v_i}}{\sum_{\forall q} e^{K_t \cdot v_q}}$ **where K_t is the value of K at the tth moment.**

  - Change the temperature, namely $1/K$, over the time.
    - ▷ *In the beginning, allow more randomness, and decrease the amount of randomness over the time.*
  - Increasing K from 0 to 5 gradually over the time does not enhance the performance [Bouzy et al'04].

Depth-i enhancement

- **Algorithm:**
  - **Enumerate all possible positions from the root after i moves are made.**
  - **For each position, use Monte-Carlo simulation to get an average score.**
  - **Use a minimax formula to compute the best move from the average scores on the leaves.**
- **Result [Bouzy et al'04]: depth-2 is worse than depth-1 due to oscillating behaviors normally observed in iterative deepening.**
  - **Depth-1 overestimates the root's value.**
  - **Depth-2 underestimates the root's value.**
  - **It is computational difficult for computer Go to get depth-i results when $i > 2$.**

Putting everything together

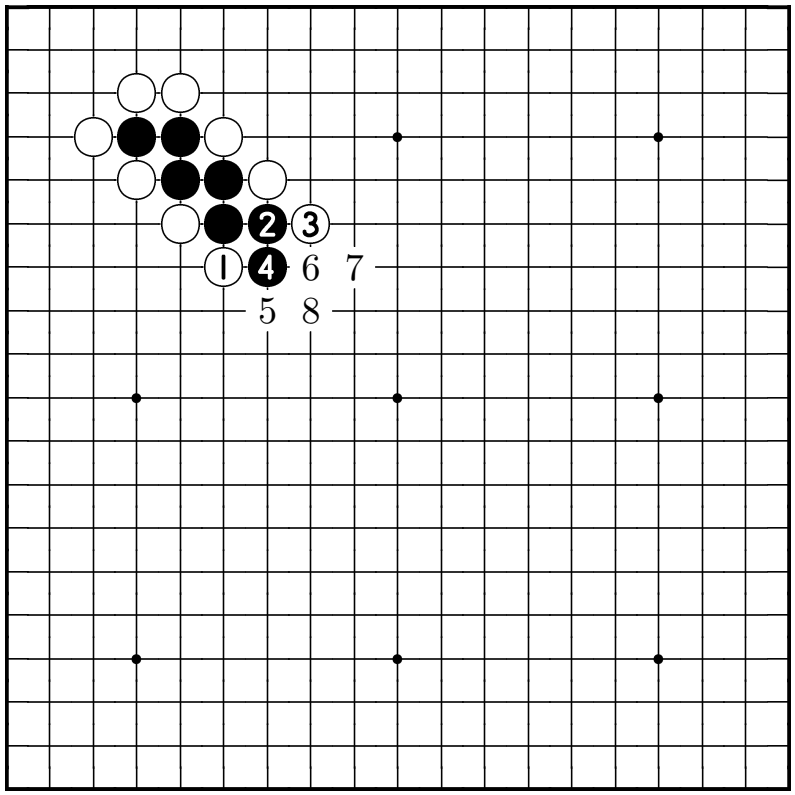
- **Two versions [Bouzy et al'04]:**
  - **Depth = 1,** $r_d = 1$**,** $\sigma_e = 0.2$ **with PP, and basic idea.**
  - $K = 2$**, no PP, and all-moves-as-first.**
- **Still worse than GnuGo in 2004, a Go program with lots of domain knowledge, by more than 30 points.**
- **Note: as we said before, most of the techniques are invented before UCT.**
  - **The idea of UCT is not part of "everything" used in his experiments.**
  - **This somehow shows that the idea of UCT may be critical among all techniques.**

Conclusions

- **Add tactical search: for example, ladders.**
  - **A ladder is a kind of string whose live-or-death is certain many plys ahead.**
- **Add more domain knowledge besides no filling of eyes: for example, in Atari, simulate extending plys first.**
  - **An extending ply is one which increases the liberty of some strings that are in Atari.**
- **As the computer goes faster, more domain knowledge can be added.**
- **Exploring the locality of Go using statistical methods.**

Ladder

- **White to move next at 1, then black at 2, then white at 3, and then black at 4, ...**



Ladder: comments

- Ladder in Go is a perfect example to illustrate the idea of getting the "right" sampling is important.
  - In the example in the last page, it is very bad for BLACK.
  - However, the WHITE only has one correct response out of a few hundreds of bad ones.
  - If you do uniform sampling, then the odds of finding the right one is remote.
- The "true" meaning of doing a "fair" random sampling is thus
  - when the position is good, do sampling so that the final outcome of a playout is more likely to be good;
  - when the position is bad, do sampling so that the final outcome of a playout is more likely to be bad.
- "Fair" sampling will be a very hard, though may not be impossible, task for a program that has no domain knowledge.
  - "Fairness" has something to do with your opponent.
    - ▷ *If your opponent is weak, then thinking too much may not be optimal.*

Comments

- We only describe some specific implementations of some general Monte-Carlo techniques.
  - Other implementations exist for say AMAF and others.
- Depending on the amount of resources you have, you can
  - decide the frequency to update the node information;
  - decide the frequency to re-pick PV_{UCB};
  - decide the frequency to prune/un-prune nodes.
- Most of the methods introduced have a statistical flavor.
  - First the heuristic is "discovered" based on some clever intuitions or observations.
  - Then people try to fine tune the parameters used in the heuristic manually.
  - Finally statistical tools are found or established to formally settle it.
- Over-use too many heuristics may cause bad side effects.
  - A warning for using the cock tail styled method.
    - ▷ *Do not know where the real contribution comes from.*
    - ▷ *Using too much resource.*

Domain dependent refinements

- **Main technique:**
  - **Adding domain knowledge.**
- **We use computer Go as an example here.**
- **Refinements come from machine learning and/or deep learning via training and prediction.**
  - **During the expansion phase:**
    - ▷ *Special case: open game.*
    - ▷ *General case: use domain knowledge to expand only the nodes that are meaningful with respect to the game considered, e.g., Go.*
  - **During the simulation phase: try to find a better simulation policy.**
    - ▷ *Simulation balancing for getting a better playout policy.*
    - ▷ *Other techniques are also known.*
- **Prediction of board evaluations, not just good moves.**
  - ▷ *Combined with UCB score to form a better estimation on how good or bad the current position is.*
  - ▷ *To start a simulation with a good prior knowledge.*
  - ▷ *To end a simulation earlier when something very bad or very good has already happened.*

Warnings

- **We only tell you how machine/deep learning are used in game playing by expecting the audience having some prior knowledge.**
- **We do not cover machine/deep learning theory, tool or implementation in this course.**

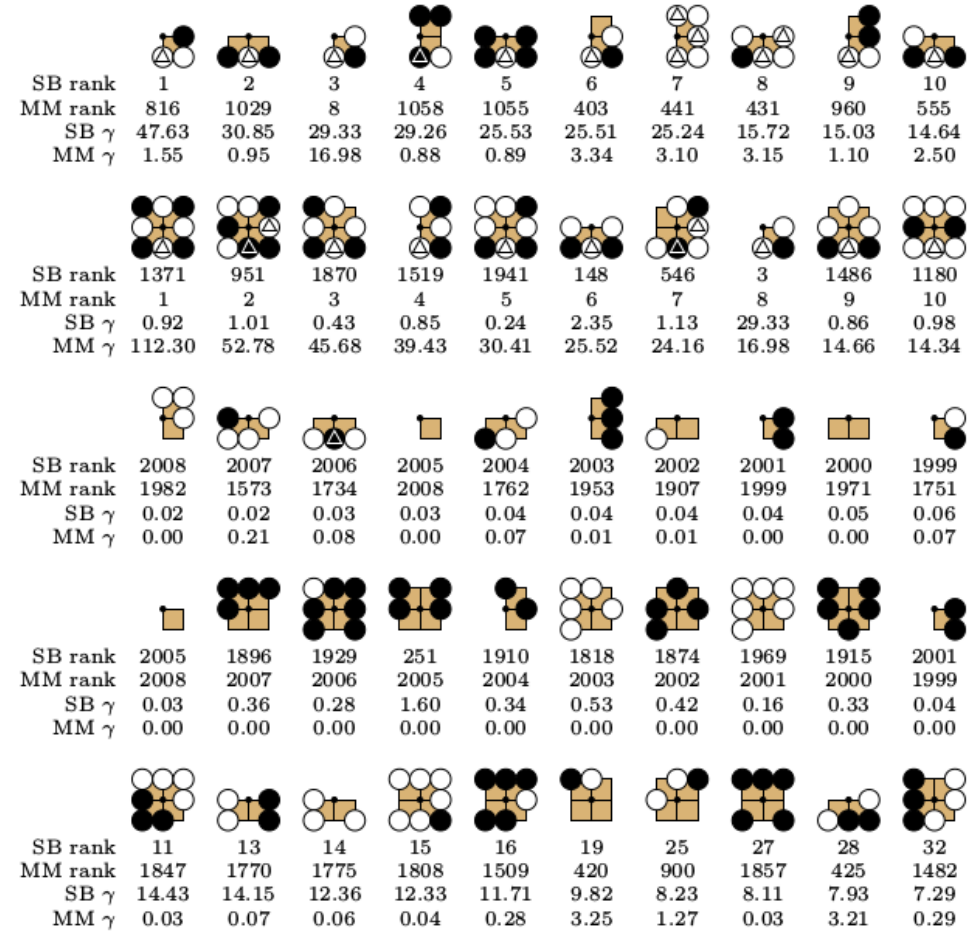
How domain knowledge can be obtained

- **Via human experts: very expensive to get and very difficult to be complete as proven by studies before year 2004 such as GNU Go.**
- **Machine learning.**
  - **(Local) pattern: treat positions as pictures and find important patterns and shapes within them.**
    - ▷ *K by K sub-boards such as $K = 3$.*
    - ▷ *Diamond shaped patterns with different widths.*
    - ▷ ...
  - **(Global) feature: find semantics of positions.**
    - ▷ *The liberties of each stone.*
    - ▷ *The number of stones can be captured by playing this intersection.*
    - ▷ ...
  - **Advanced knowledge: (higher ordered) information such as history dependent ones, namely cannot be captured using only the current position.**
    - ▷ *Ko.*
    - ▷ *Features include previous several plys of a position.*
    - ▷ *A ply that has an effect to be realized a long turns latter.*

3 by 3 patterns

- **[Huang et al'10]**

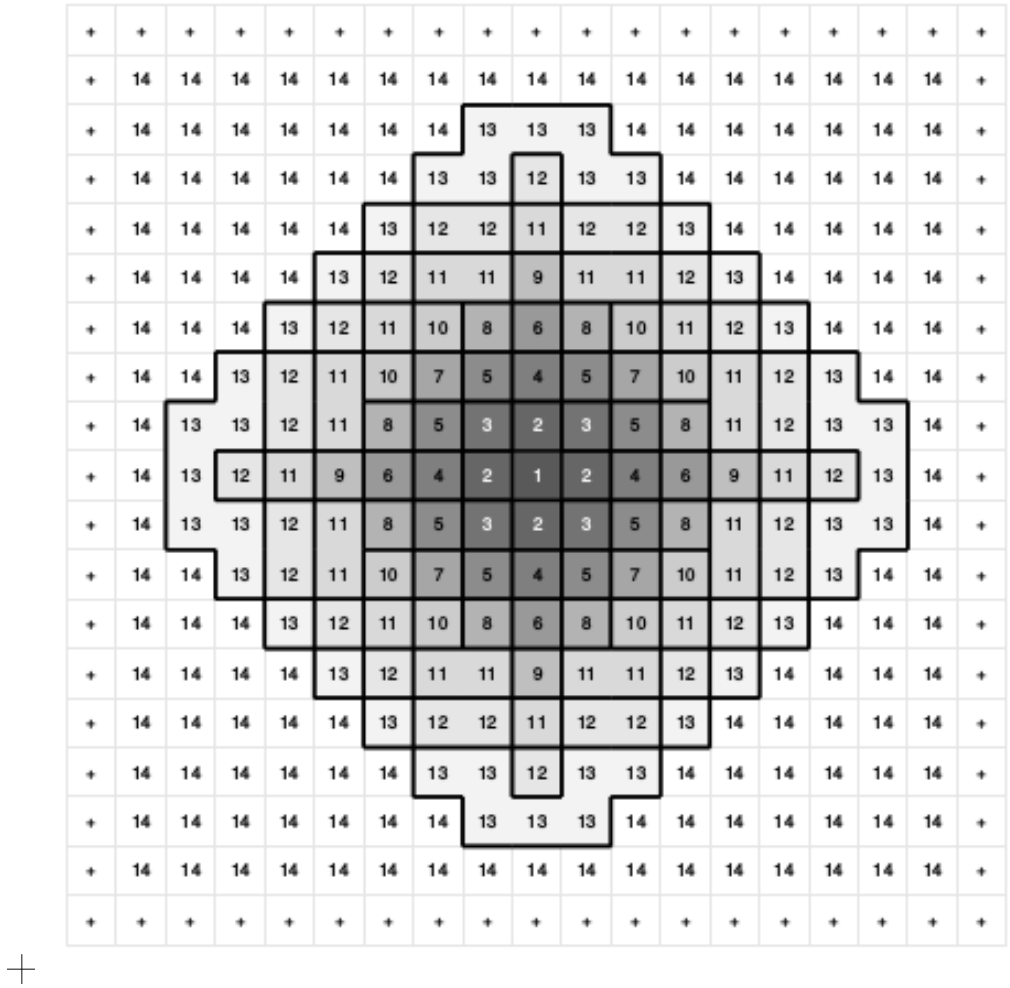
Table 4. 3×3 patterns. A triangle indicates a stone in atari. Black to move.



SB rank	1	2	3	4	5	6	7	8	9	10
MM rank	816	1029	8	1058	1055	403	441	431	960	555
SB γ	47.63	30.85	29.33	29.26	25.53	25.51	25.24	15.72	15.03	14.64
MM γ	1.55	0.95	16.98	0.88	0.89	3.34	3.10	3.15	1.10	2.50
SB rank	1371	951	1870	1519	1941	148	546	3	1486	1180
MM rank	1	2	3	4	5	6	7	8	9	10
SB γ	0.92	1.01	0.43	0.85	0.24	2.35	1.13	29.33	0.86	0.98
MM γ	112.30	52.78	45.68	39.43	30.41	25.52	24.16	16.98	14.66	14.34
SB rank	2008	2007	2006	2005	2004	2003	2002	2001	2000	1999
MM rank	1982	1573	1734	2008	1762	1953	1907	1999	1971	1751
SB γ	0.02	0.02	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.06
MM γ	0.00	0.21	0.08	0.00	0.07	0.01	0.01	0.00	0.00	0.07
SB rank	2005	1896	1929	251	1910	1818	1874	1969	1915	2001
MM rank	2008	2007	2006	2005	2004	2003	2002	2001	2000	1999
SB γ	0.03	0.36	0.28	1.60	0.34	0.53	0.42	0.16	0.33	0.04
MM γ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SB rank	11	13	14	15	16	19	25	27	28	32
MM rank	1847	1770	1775	1808	1509	420	900	1857	425	1482
SB γ	14.43	14.15	12.36	12.33	11.71	9.82	8.23	8.11	7.93	7.29
MM γ	0.03	0.07	0.06	0.04	0.28	3.25	1.27	0.03	3.21	0.29

Diamond shaped patterns

- **[Stern et al'06]**



Supervised learning

- **Use supervised learning to get a good prediction on the move to choose when a position is given: a vast amount of expert games with possible annotations are available.**
  - **Training phase.**
    - ▷ *Feed positions and their corresponding actions (moves) in expert games into the learning program.*
    - ▷ *Feature and pattern extraction from these positions.*
  - **Prediction phase.**
    - ▷ *Predict the probability of a move will be taken when a position is encountered.*
- **Many different paradigms and algorithms.**
  - **A very active research area with many applications.**

Reinforcement learning

- Use reinforcement learning to boost the baseline prediction, obtained from supervised learning for example, using self-play or expert annotated games.
  - The baseline one needs to be good enough to achieve some visible improvement.
  - Feed evaluations of positions from the baseline one into the learning program.
    - ▷ *The objective of the learning is different from the supervised learning phase.*
    - ▷ *To learn which move will result in better positions, namely positions with better evaluations.*
- Note that the predictions of moves best matched with the training data, and moves best matched with better positions may be very different.
- Many different paradigms and algorithms.
  - Another very active research area with many applications.
  - Some claimed that no baseline obtained from supervised learning is needed if the rate of convergence is fast enough [Silver et al 2017].

History

- **Using machine learning to aid computer Go programs is not new.**
  - **NeuroGo [Enzenberger'96]: neural network based move predication.**
  - **IndiGo [Bouzy and Chaslot'05]: Bayesian network.**
- **Pure learning approach is very difficult to compete with top computer Go programs with searching before AlphaGo.**
  - **Need to combine some forms of searching.**
- **Computing constraints.**
  - **It is costly, or resource consuming, to do deep learning.**
  - **In 2017, DeepMind team claimed that no supervised learning is needed even the training time is limited in training AlphaGo Zero [Silver et al 2017].**

Combining with MCTS

- **Places that MCTS needs helps.**
  - **The expansion phase: what children to explore when a leaf is to be expanded.**
  - **The simulation phase.**
    - ▷ ***Originally almost random games are generated: needs a huge amount of simulated games to have a high confidence in the outcome.***
    - ▷ ***Can we use more domain knowledge to get a better confidence using the same number of simulations?***
  - **Position evaluation: to end a simulation earlier or to start a simulation with better prior knowledge.**
- **Fundamental issue: assume we can only afford to use a fixed amount of resources R, say computing power in a given time constraint.**
  - **Assume each simulation takes r_s amount of resources for a strategy s in generating a playout.**
    - ▷ ***Hence we can only afford to have $\lfloor \frac{R}{r_s} \rfloor$ playouts.***
  - **How to pick s to maximize c_s, the confidence or quality?**
    - ▷ ***Difficult to define confidence or quality.***
  - **Not likely that r_s is linearly proportional to c_s.**

Machine learning

- **Many different framework and theories.**
  - **Decision tree.**
  - **Support vector machine.**
  - **Bayesian network.**
  - **Artificial neural network (ANN).**
  - . . .
- **Here we will only introduce Bayesian network and multi-layer artificial neural network which includes convolutional neural network (CNN) and deep neural network (DNN).**
- **For each framework, depending on how the underlying optimization problem is solved, there are many different simplified models.**
  - **We will only introduce some popular models used in game playing.**
  - **There are many open-source or public domain software available.**

Bayesian network based learning (1/3)

- **Bayes theorem:** $P(B \mid A) = \frac{P(A|B)P(B)}{P(A)}$.
  - A: features and patterns
  - B: an action or a move
  - $P(A)$: probability of A happens in the training data set
  - $P(B)$: probability of an action B is taken
  - $P(A \mid B)$: probability of A appears in the training set when an action B is taken.
    - ▷ *This is the training phase.*
    - ▷ *We can compute this information from historical records.*
  - $P(B \mid A)$: when A appears, the prediction of B is taken.
    - ▷ *This is what we want.*
- **Assume there are two actions B_1 and B_2 that one can take in a position with the feature set A, then one uses the values of $P(B_1 \mid A)$ and $P(B_2 \mid A)$ to make a decision.**
  - Take one with a larger value.
  - Take one with a chance proportional to its value.
  - Take one with a chance using the idea of simulating annealing.
  - ...

Bayesian network based learning (2/3)

- **When the training set is huge and the feature set A is large, it is very time and space consuming to compute everything exactly.**
  - **In many cases, exact computation is impossible.**
    - ▷ ***Training data are usually huge in quantity, may contain error, and most of the time incomplete.***
    - ▷ ***When there are many features in a position, it is very time and space consuming to compute $P(B \mid A)$.***
- **Use some sort of approximation.**
  - **Assume a position P is consisted of features $\mathcal{P} = \{P_{A_1}, P_{A_2}, \ldots, P_{A_w}\}$.**
  - **For a possible child position B of P, give each feature P_{A_i} a strength or influence parameter $q(B, P_{A_i})$ so that it approximates the probability of $P(B \mid P_{A_i})$.**
  - **Use a function $f(q(B, P_{A_1}), \ldots, q(B, P_{A_w}))$ to approximate the value of $P(B \mid \mathcal{P})$.**

Bayesian network based learning (3/3)

- **Many different models exist to approximate the strength or influence parameter, θ, of a party, player, feature or pattern.**
  - **Bradley-Terry (BT) model.**
    - ***Given 2 players with strengths θ_i and θ_j, $P(i \text{ beats } j) = \frac{e^{\theta_i}}{e^{\theta_i}+e^{\theta_j}}$.***
    - ***Generalized model: Comparisons between teams of players, say odds of players $i+j$ beats both $k+m$ and $j+n+p$ is $\frac{e^{\theta_i+\theta_j}}{e^{\theta_i+\theta_j}+e^{\theta_k+\theta_m}+e^{\theta_j+\theta_n+\theta_p}}$.***
  - **Thurstone-Mosteller (TM) model.**
    - ***Given 2 players with strengths that are Gaussian distributed (or normal distributed) with $\mathcal{N}(\theta_i, \sigma_i^2)$ and $\mathcal{N}(\theta_j, \sigma_j^2)$, $P(i \text{ beats } j) = \Phi(\frac{e^{\theta_i}-e^{\theta_j}}{\sqrt{\sigma_i^2+\sigma_j^2}})$, where $\mathcal{N}(\mu, \sigma^2)$ is a normal distribution with mean μ and variance σ^2, and Φ is the c.d.f. of the standard normal distribution, namely $\mathcal{N}(0, 1)$.***
    - ***Generalized TM model is more involved.***
- **May not be reasonable in real life.**
  - **Does not allow cyclic relations among players.**
  - **Strength of a team needs not to be product of teammate's strength.**
- **We will use mainly BT model to illustrate the ideas here.**

BT model in computing Elo

- **This is also how Elo rating system is computed between players in games like Chess or Go.**
  - **Example: The Elo rating number of player i with strength θ_i is $400 \log_{10}(\theta_i)$.**
    - ***Assume the Elo ratings of players A, B and C are 2,800, 2,900 and 3,000 respectively.***
    - $P(C \textbf{ beats } B) = \frac{10^{3000/400}}{10^{3000/400}+10^{2900/400}} = \frac{10^{7.5}}{10^{7.5}+10^{7.25}} \sim 0.64.$
    - $P(B \textbf{ beats } A) = \frac{10^{2900/400}}{10^{2900/400}+10^{2800/400}} = \frac{10^{7.25}}{10^{7.25}+10^{7}} \sim 0.64.$
    - $P(C \textbf{ beats } A) = \frac{10^{3000/400}}{10^{3000/400}+10^{2800/400}} = \frac{10^{7.5}}{10^{7.5}+10^{7}} \sim 0.76.$
    - ***Note that $P(i$ beats $j) + P(j$ beats $i) = 1$ by assuming no draw.***
  - **Note: In Elo system, we use $10^{\theta_i/400}$ not e^{θ_i}.**
- **Fundamental problem:**
  - **When you have a huge set of data, how to compute the strength parameters using limited amount of resources?**
  - **The problem is even bigger when data may contain some errors and/or incomplete.**

Minorization-Maximization (MM)

- **Minorization-Maximization (MM): an approximation algorithm for the BT model [Coulom'07].**
  - **Patterns: all possible, for example $3 * 3$ patterns, i.e., $3^9 = 19,683$ of them [Huang et al'11].**
  - **Training set: records of expert games.**
- **During the simulation phase, use the prediction algorithm to form a random playout by finding average next moves.**
  - **It is easy to have an efficient implementation.**
  - **Can add some amount of randomness in selecting the moves, such as using the idea of temperature in simulated annealing.**
- **Results are very good: 37.86% correctness rate using 10,000 expert games [Wistuba et al'12]**
  - **A very good playout policy may not be good enough for the purpose of finding out the average behavior.**
    - ▷ ***The samplings must consider the average "real" behavior of a player can make.***
    - ▷ ***It is extremely unlikely that a player will make trivially bad moves.***
  - **Need to balance the amount of resources used in carrying out the policy found and the total number of simulations can be computed.**

Simulation balancing (SB)

- **Use the idea of self-play games to boost the performance [Huang et al'11].**
  - **Supervised plus reenforcement learning.**
  - **Feature set can be smaller.**
  - **Normally does not learn what moves are played in expert games, but how good or bad a position is.**
    - ▷ *Some forms of position evaluation.*
- **Results are extremely positive for 9 by 9 Go.**
  - **Against GNU Go 3.8 level 10.**
    - ▷ 59.3% *winning rate using MM against a good baseline of* 50%*.*
    - ▷ 62.6% *winning rate using SB against a good baseline program of* 50%*.*
- **Results are not as good on 19 by 19 Go against one using MM along.**
- **Erica, a computer Go program, later improved the SB ideas in [Huang et al'11] won 2010 Computer Olympiad 19x19 Go Gold medal.**

How they were used then

- **Assume using the BT model.**
- **Generation of the pattern database:**
  - **Manually construct.**
  - **Exhaustive enumeration: small patterns such as 3 by 3.**
  - **Find patterns happened more than a certain number of times in the training set.**
    - ▷ ***Patterns, for example diamond-shapes, are too large to enumerate.***
- **Training.**
- **Setting of the parameters:**
  - **Assume after training, feature or pattern i has a strength θ_i.**
  - **Let the current position be P with b possible child positions $P_1, \ldots, P_b$.**
  - **Let F_i be the features or patterns occurred in P_i.**
  - **Let the score of P_i be $S_i = \Pi_{j \in F_i} \theta_j$.**
- **Child P_j is chosen with the probability $\frac{S_j}{\sum_{i=1}^{b} S_i}$.**
  - ▷ ***The best child is one with the largest score.***

Comments

- **Implementation:**
  - **Incrementally update the features and patterns found.**
  - **Use some variations of the Zobrist hash function to efficiently find the strength of a feature or pattern.**
- **We only show two possible avenues of using Bayesian network based learning via using the BT model, namely MM and SB.**
  - **There are many other choices such Bayesian full ranking.**
- **The training phase needs to be done once, but takes a huge amount of space and time.**
  - **Usually use some forms of iterative updating algorithms to obtain the parameters, namely the strength vector, of the model.**
  - **For MM with k distinct features or patterns, n training positions and an average b legal moves for each position, it takes $O(kbn)$ space, and X iterations each of which takes $O(bnkh + k^2bn)$ time, where h is the size of the pattern or feature and X is the number of iterations needed for the approximation algorithm to converge [Coulum'07].**
- **The prediction phase takes only $O(kh)$ space and time.**
- **Q: Can the part of feature extraction and weights of multiple features be done automatically?**

Artificial Neural Network

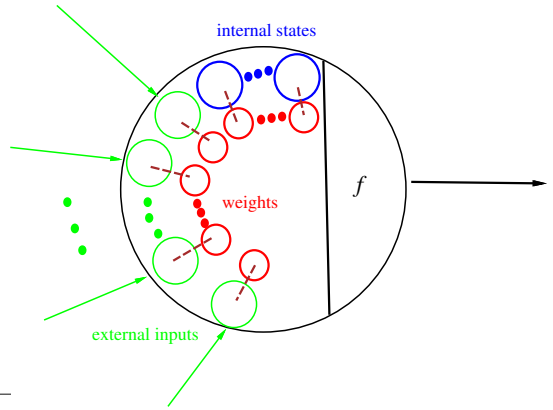
- **Using a complex network of neurons to better approximate non-linear optimizations.**
  - **Usually called deep learning when the number of artificial neural network layers is more than 1.**
  - **Can have different architectures such as CNN or DNN.**
- **A hot learning method inspired by the biological process of the animal visual cortex.**
  - **Each neuron takes input from possibly overlaid neighboring sub-images of an image, and then assigns appropriate weights to each input plus some values within the cell to compute the output value.**
  - **This process can have multiple layers, namely a neuron's output can be other neurons' inputs, and forms a complex network.**
  - **Depending on the network structure, Bayesian network approaches tends to need less resources than the CNN or DNN approach.**
  - **There are also training phase and prediction phase.**
- **Many different tools which can be parallelized using GPU.**
  - **Need a great deal of resources to do training and some amount of time to do prediction.**

Basics of ANN (1/3)

- **Assume the ith neuron whose output is z_i takes m_i inputs $x_{i,1}, \ldots, x_{i,m_i}$, and has internal states $y_{i,1}, \ldots, y_{i,n_i}$.**
  - **We want to assign weights $w_{i,1}, \ldots, w_{i,m_i+n_i}$ so that**

$$z_i = f((w_{i,1} * x_{i,1}), ..., (w_{i,m_i} * x_{i,m_i}), (w_{i,m_i+1} * y_{i,1}), ..., (w_{i,m_i+n_i} * y_{i,n_i})),$$

  where f is a transformation/activation function that is not hard to compute.
- **Neurons are connected as a inter-connection network where outputs of neurons can be inputs of others.**



Basics of ANN (2/3)

- **Sometime for simplicity**

$$z_i = f(\sum_{j=1}^{m_i}(w_{i,j} * x_{i,j}) + \sum_{j=1}^{n_i}(w_{i,j+m_i} * y_{i,j})).$$

- **f is often called activation function that normalize the value.**
  - **Examples:**
    - ***Binary step:*** $f(x) = (x \leq 0)?0 : 1$
    - ***ReLU (Rectified Linear Unit):*** $f(x) = (x < 0)?0 : x$
    - $\ldots$
  - **Desired properties in obtaining optimization and consistence:**
    - *Nonlinear*
    - *Continuously differentiable*
    - *Monotonic*
    - $\ldots$

Basics of ANN (3/3)

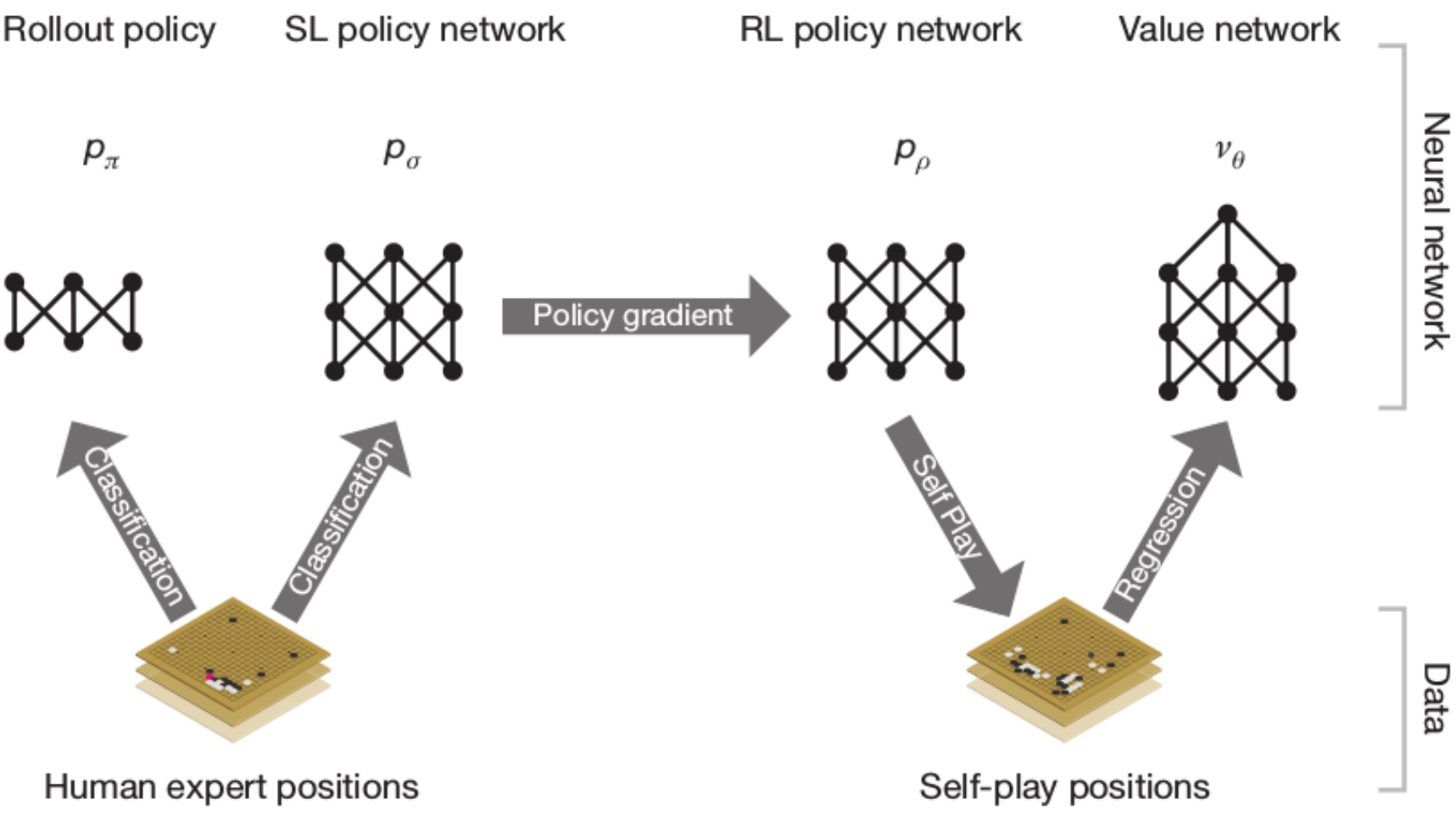
- **Measurement of success**
  - **Accuracy: the percentage of your predicted values equal to their actual values.**
    - ▷ *Accuracy may not be a good indicator of success since not all events, for example false positive and false negative, are equal.*
    - ▷ *Example: assume a rare event happened in a training set, then answering all negative's gives you a high accuracy, but useless prediction.*
- **When there are multiple input data set, we want to find an assignment of the weights so that some loss or error function is minimized.**
  - **The loss or error function can be the average distance, in terms of L_1 or L_2 metric, among the training data set.**
  - **May want to use some log scale such as cross entropy.**
- **Many different algorithms exist to compute approximated values for the weights.**
  - **Computation time intensive.**
  - **Space usage intensive.**

Deep learning

- **Use artificial neural network of different sizes and structure to achieve different missions in playing 19 by 19 Go [Silver et al'16].**
  - **Supervised learning (SL) in building policy networks which spell out a probability distribution of possible next moves of a given position.**
    - ▷ ***A fast rollout policy:** for the simulation phase of MCTS, prediction rate is 24.2% using only 2 μs.*
    - ▷ ***A better SL rollout policy:** 13-layer CNN with a prediction rate of 57.0% using 3 ms.*
  - **Reinforcement learning (RL): obtain both a better, namely more accurate, policy network and at the same time a value network for position evaluation.**
    - ▷ ***RL policy:** further training on the top of the previously obtained SL policy using more features and self-play games that achieves an 80% winning rate against the SL rollout policy.*
    - ▷ ***Value network:** using the RL policy to train for knowing how good or bad a position is.*

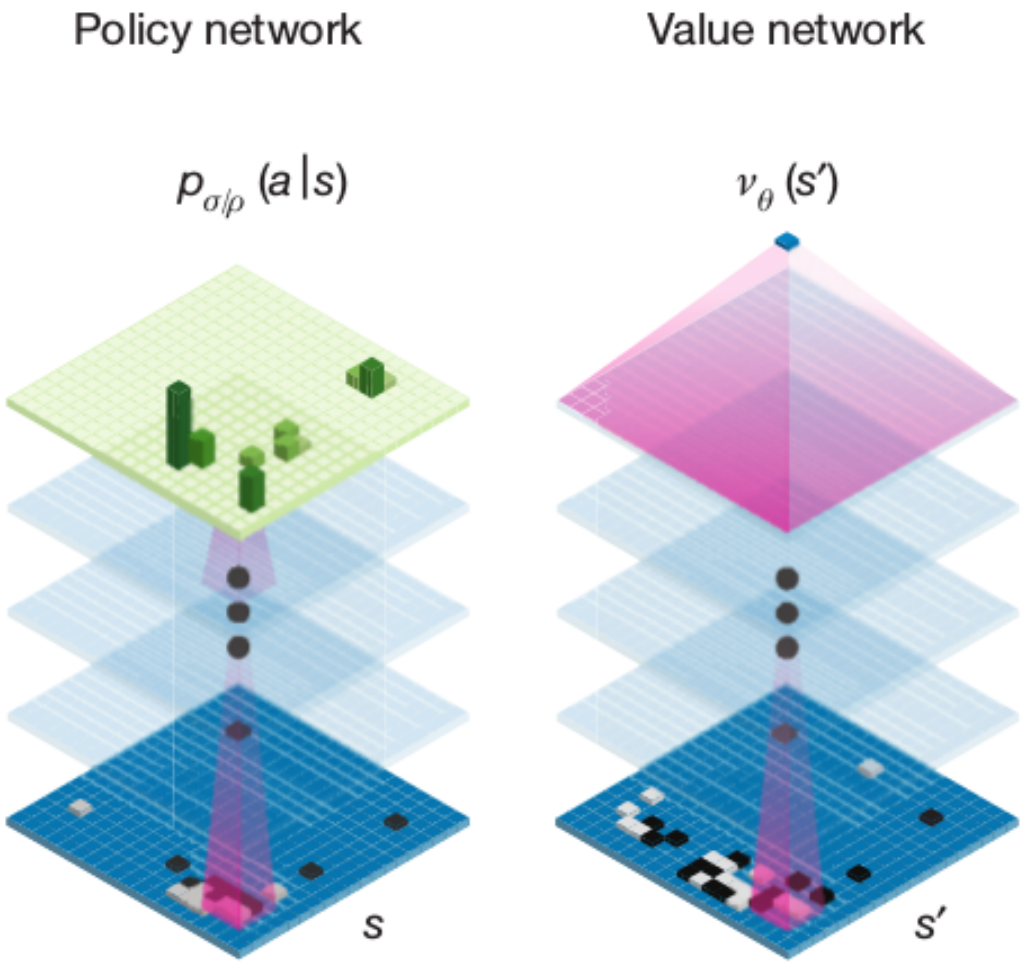
Various networks in AlphaGo

- **[Silver et al'16]**



How networks are obtained by AlphaGo

- **[Silver et al'16]**

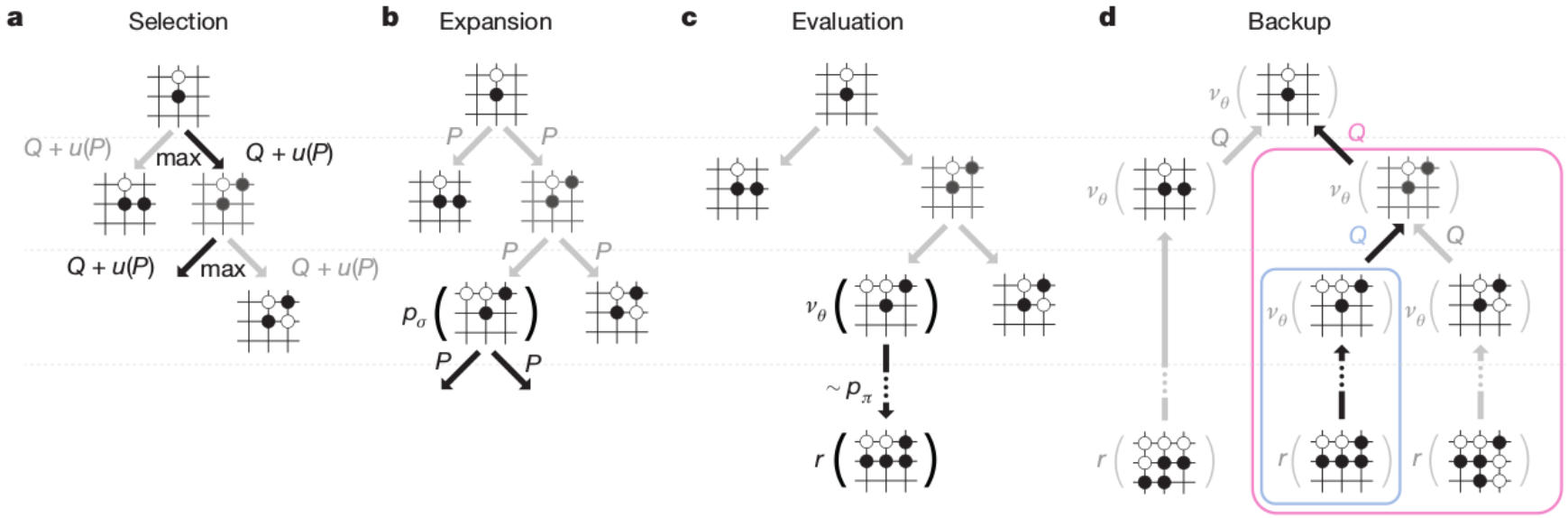


Combining networks

- **Use a fast, but less accurate, SL Rollout policy to do the simulations.**
  - **Need to do lots of simulations.**
- **Use a slow, but more accurate, SL policy in the expansion phase.**
  - **Do not need to do node expansions too often.**
- **Use a slow, resource consuming and complex, but more informatic RL policy to construct the value network.**
  - **Do not need to do node evaluations too often.**
- **Using a combination of the output from the value network and the current score from the simulation phase, one can decide whether to end a simulation earlier or not.**

How networks are used in AlphaGo

- **[Silver et al'16]**



Comments (1/3)

- **A very good tradeoff in performance and amount of resources used.**
  - **A less accurate but fast rollout policy is used with MCTS so that in the tree search part the correctness rate can be increased.**
    - ▷ *Need to do lots of simulations so each cannot take too much time.*
  - **Use a slow but more accurate policy for tasks such as expansion that do not need to carry out many times.**
  - **Use reinforcement learning in obtaining a value network to replace the role of designing complicated evaluating functions.**
- **Now is the way to go for computer Go!**
  - **Performance is extremely well and is generally considered to be over human champion.**
  - **Lots of legacy teams such as Zen and Crazystone are embracing ANN.**
  - **New teams such as Darkforest developed by Facebook, Fine Art developed by Tencent, and CGI developed by NCTU Taiwan, are catching up before 2016.**
- **Latest (after 2017)**
  - ▷ *Darkforest has turned open sourced in 2016.*
  - ▷ *After the introducing of AlphaGo Zero in 2017, it is difficult for others to catch up.*

Comments (2/3)

- This approach can be used in many applications such as medical informatic which includes medical image and signal reading.
  - Anything that is pattern related and has lots of data collected with expert annotations.
- Take a lot of computing resources for computer Go.
  - More than 100,000 features and patterns.
  - More than 40 machines each with 32 cores and a total of more than 176 GPU cards whose power consumption is estimated to be in the order of 10^3 KW.
  - AlphaGo Zero claims to use much less resources.
- More studies are needed to lower the amount of resources used and to do transfer learning, namely duplicate the successful experience on one domain to another domain.

Comments (3/3)

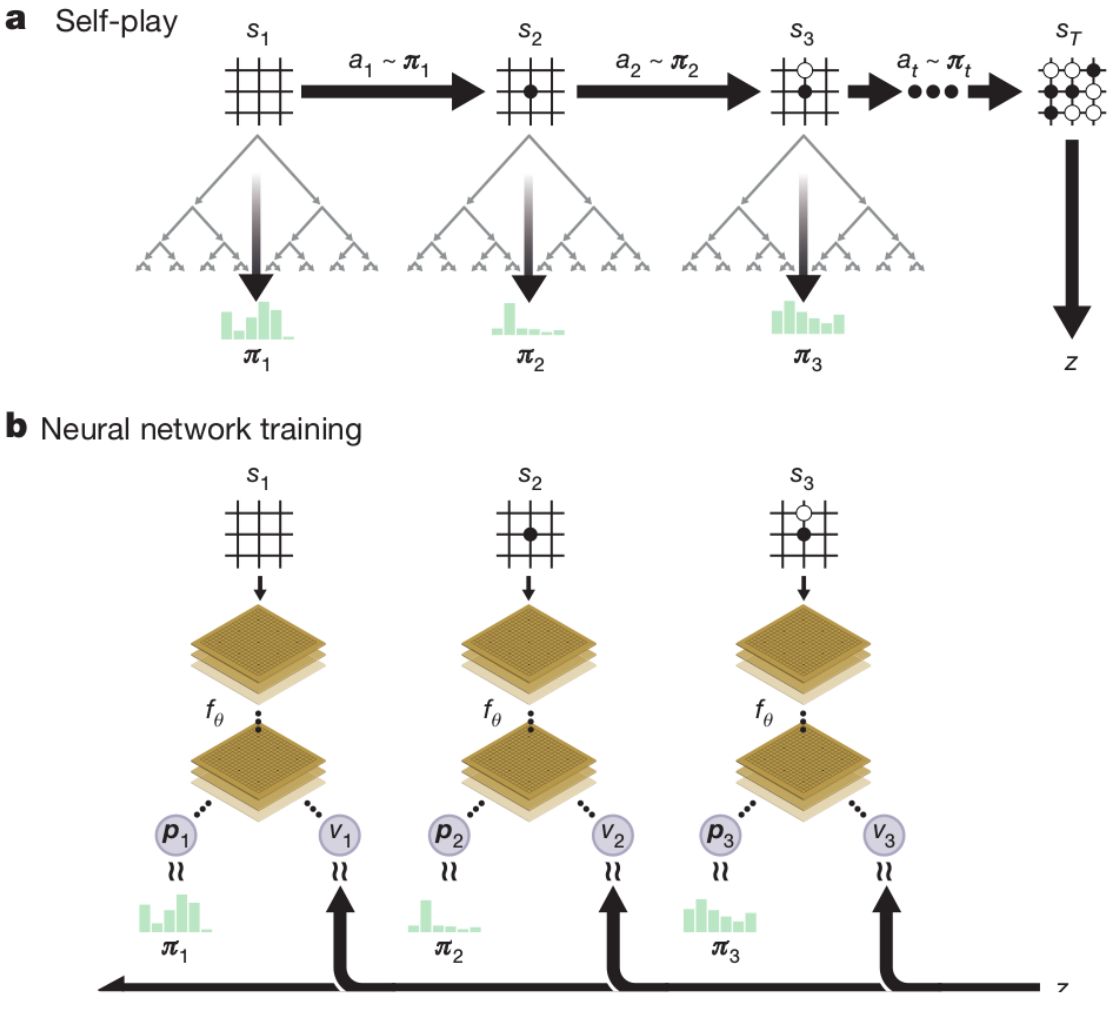
- We only know it works by building the ANN, but it is almost impossible to explain how it works.
  - Very difficult to debug if a silly bug occurs.
  - Very difficult to "control" it to act the way you want/expect to.
  - It is an art to find the right coefficients and tradeoff.
- We also describe some fundamental techniques and ideas in the part of combining machine learning.
  - Other machine learning tools are also available and used.
- Using machine learning or MTCS along won't solve the performance problem in computer Go. However, the combination of both does the magic.

AlphaGo Zero

- **Latest result: AlphaGo zero uses no supervised learning to achieves the top of computer Go at an Elo rating of 5185 [Silver et al. 2017].**
- **Main methods:**
  - **Trained solely by self-play reinforcement learning, starting from random play, without any supervision or use of human data.**
  - **Uses only the black and white stones from the board as input features.**
  - **Uses a single neural network, rather than separate policy and value networks.**
  - **Uses a simpler tree search that relies upon this single neural network to evaluate positions and sample moves, without performing any Monte Carlo rollouts.**
- **Contribution:**
  - **A new reinforcement learning algorithm that incorporates lookahead search inside the training loop, resulting in rapid improvement and precise and stable learning.**

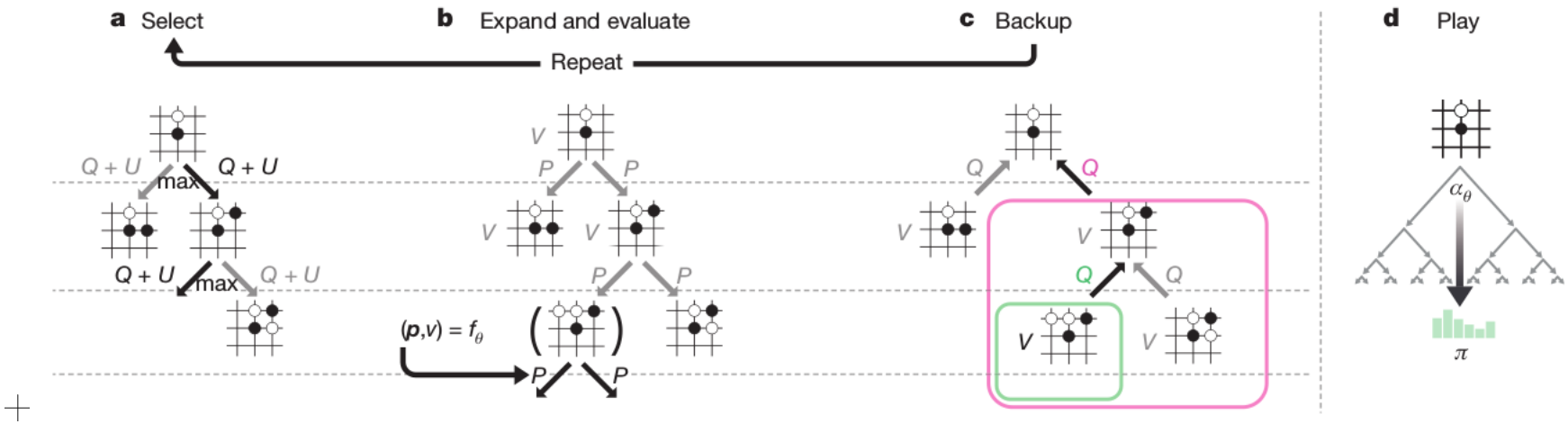
Training while self-playing

- **[Silver et al'17]**



MCTS and training together

- **[Silver et al'17]**



Comments

- Updating the network each ply you do in a self-play.
- Fast stabilizing in just 72 hours.
- Helped by special hardware and the total power consumption is greatly reduced.
  - A single machine with 4 TPU's.
- Is this a unique experience or something can be used in many other applications?

Alpha Zero (1/2)

- **A deep learning program to end all programs in game playing!**
  - Starting from random play and given no domain knowledge except the game rules, Alpha Zero is a general algorithm that masters Chess, Go and Shogi.
    - ▷ *No need to do supervised learning.*
    - ▷ *MCTS with deep learning beats alpha-beta with a human tuned evaluating function.*
  - Claim to be as well for games with less-defined rules.
    - ▷ *I tend to believe this is true!*
  - "AlphaZero shows that it can learn that knowledge automatically – at least if you have Google's 5,000 TPUs, which is a lot of computing!" — Daylen Yang (a member of the Stockfish Chess program team) [Stetka'18].
- The Facebook PolyGames project tried to do a similar magic.

Alpha Zero (2/2)

- **Papers**
  - ***Mastering Chess and Shogi by Self-Play with a General Reinforcement Learning Algorithm.***
    David Silver, Thomas Hubert, Julian Schrittwieser, Ioannis Antonoglou, Matthew Lai, Arthur Guez, Marc Lanctot, Laurent Sifre, Dharshan Kumaran, Thore Graepel, Timothy Lillicrap, Karen Simonyan, Demis Hassabis.
    arXiv:1712.01815, Dec. 5, 2017.
  - ***A general reinforcement learning algorithm that masters chess, shogi, and Go through self-play***
    David Silver, Thomas Hubert, Julian Schrittwieser, Ioannis Antonoglou, Matthew Lai, Arthur Guez, Marc Lanctot, Laurent Sifre, Dharshan Kumaran, Thore Graepel, Timothy Lillicrap, Karen Simonyan, Demis Hassabis.
    Science, 07 Dec 2018: 1140-1144.
  - ***"Superhuman" AI Triumphs Playing the Toughest Board Games***
    Bret Stetka
    Scientific American, December 6, 2018.

Comments (1/2)

- **What can be done after Alpha Zero?**
  - **It is better to know "what cannot be done using Alpha Zero?"**
- **Beating the best human is only a very small, yet glorious, part of why we started doing Computer game playing researches.**
  - **A machine beats a human in a certain skill is predictable and inevitable for any skill that can be defined by a limited set of rules.**
  - **Machines, with a perfect duplicating capacity, are faster and more resourceful every day, but human only gets older and fade away.**
- **The real purpose of playing games using computers.**
  - **Enabling computers more useful to human.**
    - ▷ *Programming and problem solving skills that can be used in other areas.*
    - ▷ *Helping human to have a better life.*
  - **Understanding fundamental structures and properties of games.**
    - ▷ *What properties do a game have?*
      Fairness Fun Educational Boundary effects
    - ▷ *What rules or designs to make a game having such properties?*
    - ▷ *Why this position is more difficult to human than others?*
    - ▷ *...*

Comments (2/2)

- **Skills with limited pre-defined rules, including low-level programming, are going to fade away.**
  - **Deep learning models are built for lots of complicated previously unsolvable exactly algorithms.**
  - **More examples:**
    - ***What used to need coding in assembly/machine languages to achieve desirable performance 30 years ago are now replaced by high-level programming languages and maybe sometimes programming languages running in virtual machines or by interpreters.***
    - ***What used to need coding for simple accounting functions 20 years ago are now replaced by simple spreadsheet softwares like EXCEL.***
    - ***What are provided in the std:: library of C++17 are hard codings for most programmers before the year 2011.***
    - ***Python, PHP, C# ...***
- **What is your role, as a human being, in the age of AI emerging or "disrupting"?**
  - **Are we at least part of the revolution or evolution?**

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