Theory of Computer Games: An A.I. Oriented Introduction

Tsan-sheng Hsu

徐讚昇

tshsu@iis.sinica.edu.tw

http://www.iis.sinica.edu.tw/~tshsu

Abstract

- **The link between researches in games and those in A.I.**
- **Intelligence**
- **History**
- **Definitions and characteristics**
- **Concluding remarks**

A.I. and game playing

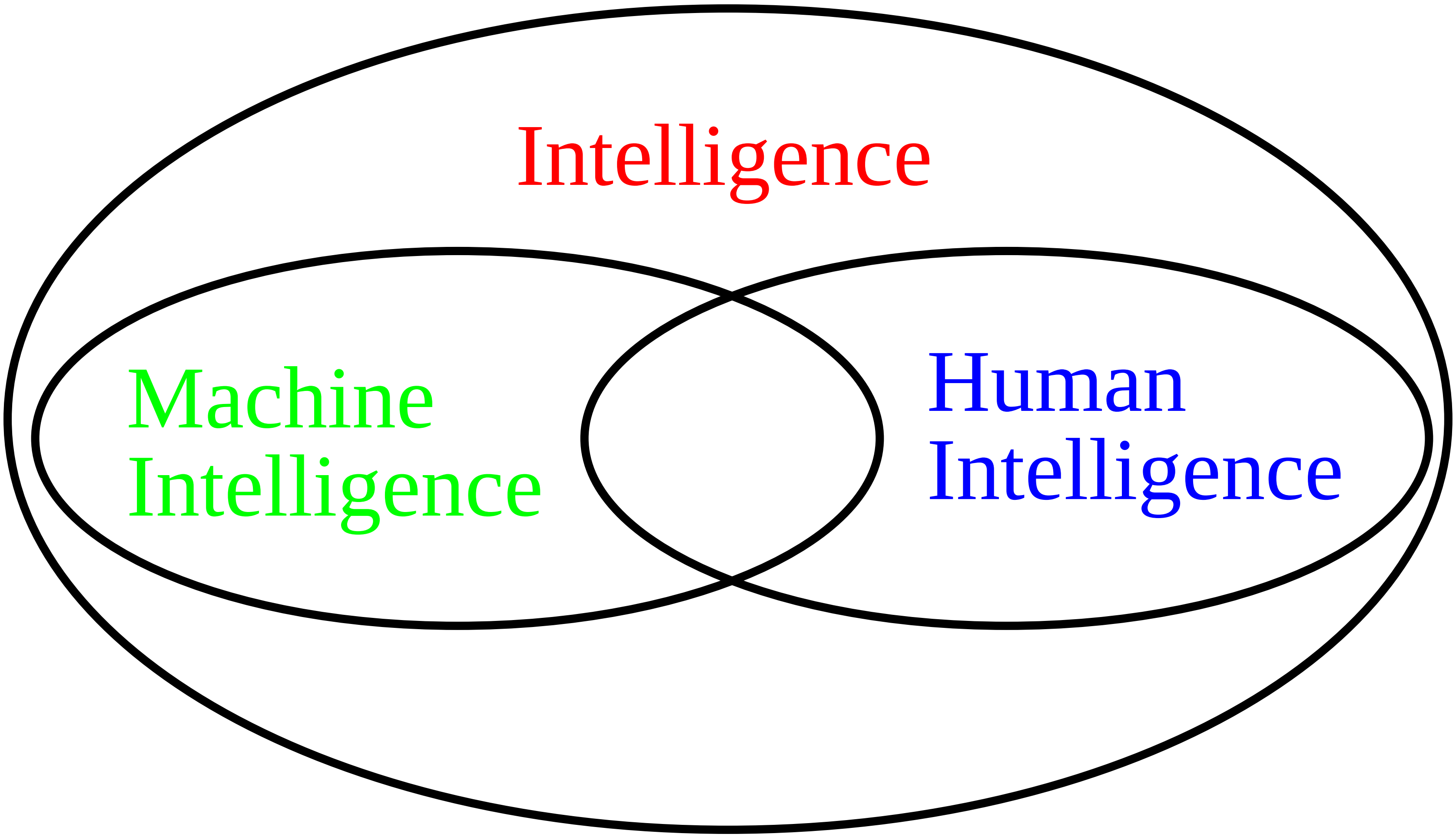
- **Patrick Henry Winston 1984 [Win84].**
  - *Artificial Intelligence* (A.I.) is the study of ideas that enable computers to be intelligent.
  - One central goal of A.I. is to make computers more useful (to human beings).
  - Another central goal is to understand the principles that make intelligence possible.
    - ▷ *Making computers intelligent helps us understand intelligence.*
    - ▷ *Intelligent computers are more useful computers.*
- **Elaine Rich 1983 [Ric83].**
  - Intelligence requires *knowledge.*
  - Games hold an inexplicable fascination for many people, and the notion that computers might play games has existed at least as long as computers.
  - Reasons why games appeared to be a good domain in which to explore machine intelligence.
    - ▷ *They provide a structured task in which it is very easy to measure success or failure.*
    - ▷ *They did not obviously require a large amount of knowledge.*

Intelligence – Turing Test

- **How to define intelligence?**
  - **Very difficult to formally define "intelligence."**
  - **Imitation of human behaviors.**
- **The Turing test**
  - **If a machine is intelligent, then it cannot be distinguished from a human [SCA03].**
    - ▷ ***Use this feature to filter out computer agents for online systems or online games.***
    - ▷ ***CAPTCHA: Completely Automated Public Turing test to tell Computers and Humans Apart***
    - ▷ ***It is a good test if designed "intelligently" to distinguish between human and non-human.***
  - **Loebner Prize Contest Yearly.**
  - **Recent advances of A.I. chat agents using deep learning, facts gathered on the web, and on-line feedback.**
  - **It may be easier now to fool people since we have all the nice interfaces built such as VR.**

Problems of Turing Test

- **Are all human behaviors intelligent?**
- **Can human perform every possible intelligent behavior?**
- **Human intelligence $=?=$ Intelligence.**
- **Can some forms of intelligence be bad for human?**
  - **Some forms of "intelligence" shown from being able to store and retrieve efficiently massive amount of data which human cannot reproduce easily.**
- **Comments: To me intelligence seems to require at least data, computation, and a good balancing usage of the above two.**
  - **May be something else!**
    - ▷ *Some forms of parallel pattern recogniztion abaility as shown in GPU based computing.*
  - **They are also the focus points of computer games research.**


Intelligence
Machine Intelligence
Human Intelligence

Shifting goals

- **From Artificial Intelligence to *Machine Intelligence*.**
  - **Lots of things can be done by either human and machines.**
  - **Something maybe better be done by machines.**
  - **Some other things maybe better be done by human.**
  - **Try to get the best out of every possible worlds!**
- **From imitation of human behaviors to doing intelligent behaviors.**
- **From general-purpose intelligence to domain-dependent *Expert Systems*.**
- ***From solving games, to understand intelligence, and then to have fun.***
  - ▷ ***Recreational***
  - ▷ ***Educational***

Early ages: The Maelzel's Chess Automaton

- **Late 18th century.**
  - **The *Turk* [LN82].**
  - **Invented by a Hungarian named Von Kempelen ($\sim$ 1770 AD).**
  - **Chess-playing "machine."**
    - ▷ *Operated by a concealed human chess-master.*
  - **"Arguments" made by the famous writer Edgar Allen Poe in *"Maelzel's Chess Player"*.**
    - ▷ It is as easy to design a machine which will invariably win as one which wins occasionally.
    - ▷ Since the Automaton was not invincible it was therefore operated by a human.
  - **Burned in a fire at an USA museum (year 1854).**
  - **"Recently" (year 2003) reconstructed in California, USA.**

Early ages: Endgame chess-playing machine

- **1912**
  - **Made by Torres y Quevedo.**
    - ▷ *El Ajedrecista (The Chess Player) [McC04]*
    - ▷ *Debut during the Paris World Fair of 1914*
  - **Plays an endgame of king and rook against king.**
  - **The machine played the side with king and rook and would force checkmate in a few moves however its human opponent played.**
  - **An explicit set of rules are known for such an endgame.**
  - **Very advanced automata for that period of time.**

Early ages: China

- **Not much materials can be found (by me)!**
  - **Some automatic machines in human forms for entertainments.**
  - **Not much for playing “games”.**
- **Shen, Kuo, (沈括 夢溪筆談) (∼ 1086 AD)**
  - **Analyzed the state space of the game Go.**
  - 卷十八
    - ▷ 小說：唐僧一行曾算棋局都數，凡若干局盡之。余嘗思之，此固易耳，但數多，非世間名數可能言之，今略舉大數。凡方二路，用四子，可變八十一局，方三路，用九子，可變一萬九千六百八十三局。方四路，用十六子，可變四千三百四萬六千七百二十一局。方五路， ... 盡三百六十一路，大約連書「萬」字四十三，即是局之大數。 ...
    - ▷ 其法：初一路可變三局，一黑、一白、一空。自後不以橫直，但增一子，即三因之。凡三百六十一增，皆三因之，即是都局數。...
    - ▷ 又法：先計循邊一行為「法」，凡十九路，得一十億六千二百二十六萬一千四百六十七局。凡加一行，即以「法」累乘之，乘終十九行，亦得上數。
    - ▷ 又法：以自「法」相乘，得一百三十五兆八百五十一萬七千一百七十四億四千八百二十八萬七千三百三十四局，此是兩行，凡三十八路變得此數也。下位副置之，以下乘上，又以下乘下，置為上位；又副置之，以下乘上 ，以下乘下；加一「法」，亦得上數。
    - ▷ 有數法可求，唯此法最徑捷。只五次乘，便盡三百六十一路。千變萬化，不出此數，棋之局盡矣。

Contributions (1/2)

- Define a cell has 3 different states.
  - 其法：初一路可變三局，一黑、一白、一空。
  - Black, white and empty.
- The state space is tripled by adding one cell.
  - 自後不以橫直，但增一子，即三因之。
  - The state space of an $n \times n$ Go board is thus 3^{n^2}.
- Algorithms to compute 3^N while $N = n \times n$ can be very large.
  - Naive algorithm (iterative):
    - ▷ *Needs $N - 1 = O(n^2)$ multiplications.*
  - Another algorithm (memorizing, divide and conquer):
    - ▷ 又法：先計循邊一行為「法」，凡加一行，即以「法」累乘之...
    - ▷ *First compute $X_0 = 3^n$ which is called* 「法」.
    - ▷ *Then compute X_0^n.*
    - ▷ *Needs $O((n-1)+(n-1))$ multiplications.*
  - Squaring algorithm (repeatedly squaring):
    - ▷ 又法：以自「法」相乘，...
    - ▷ *We know $3^i \times 3^i = 3^{2i}$.*
    - ▷ *Needs $O(\log_2 N = 2\log_2 n)$ multiplications.*

Contributions (2/2)

- **Note: the squaring algorithm takes care of the case when N is not a power of 2.**
  - ▷ 下位副置之，以下乘上，又以下乘下，置為上位；又副置之，以下乘上，以下乘下；加一「法」...
  - ▷ *First compute* $X_0 = 3^n$.
  - ▷ *Then compute* $X_1 = X_0 * X_0 = 3^{2\times n}$.
  - ▷ *Then compute* $X_2 = X_1 * X_1 = 3^{4\times n}$.
  - ▷ *Then compute* $X_3 = X_2 * X_2 = 3^{8\times n}$.
  - ▷ *Then compute* $X_4 = X_3 * X_3 = 3^{16\times n}$.
  - ▷ *Then compute* $Z = X_4 * X_1 * X_0 = 3^{19\times n}$.
- **Comments**
  - ▷ 只五次乘 $\rightarrow$ 只六次乘
  - ▷ 加一「法」 $\rightarrow$ 加乘一「法」
- **Squaring is an optimal algorithm.**
  - ▷ 有數法可求，唯此法最徑捷。
  - ▷ *Probably the earliest written record of "analysis of algorithms."*

Heritage of Computer Games Research (1/2)

- **Computer games are studied by the founding fathers of Computer Science**
  - **J. von Neumann, 1928, "Math. Annalen" [Neu28]**
  - **C.E. Shannon, 1950, Computer Chess paper [Sha50]**
  - **Arthur Samuel began his 25-year quest to build a strong checkers-playing program at 1952 [Sam60]**
    - ▷ ***In this paper, the term "machine learning" is first formed.***
      A. L. Samuel, "Some studies in machine learning using the game of checkers," in IBM Journal of Research and Development, vol. 44, no. 1.2, pp. 206-226, 1959, doi: 10.1147/rd.441.0206.
  - **Alan Turing, 1953, chapter 25 of the book "Faster than thought", entitled "Digital Computers Applied to Games" [TBBS53]**
    - ▷ ***A human "simulation" of a chess algorithm given in the paper.***

Heritage of Computer Games Research (2/2)

- **Computer games are also studied by great names of Computer Science who may not seem to have a major contribution in the area of Computer games or A.I.**
  - **D. E. Knuth (1979, compiling theorems)**
  - **K. Thompson (1983, Unix O.S.)**
  - **B. Liskov (2008, Object-oriented programming)**
  - **J. Pearl (2011, Bayesian networks)**

History (1/5)

- **Early days: A.I. was plagued by over-optimistic predictions.**
  - **Mini-Max game tree search**
  - **Alpha-Beta pruning**
- **1970's and 1980's.**
  - **Concentrated on Western chess.**
  - **Brute-force approach.**
    - *The CHESS series of programs [SA83] by the Northwestern University: CHESS 1.0 (1968), . . ., CHESS 4.9 (1980)*
  - **Theoretical breakthrough: Analysis of Alpha-Beta pruning by Knuth and Moore in 1975 [KM75].**
  - **Building faster search engines.**
  - **Chess-playing hardware.**
- **Early 1980's until 1990's.**
  - **Advances in theory of heuristic searches.**
    - *Scout, NegaScout, Proof number search*
    - *Search enhancements such as null moves and singular extensions*
    - *Machine learning*

History (2/5)

- **1990's**
  - **Parallelization and construction of massive offline databases.**
  - **Witness a series of dramatic computer successes against the best of humanity.**
    - ▷ *CHINOOK, checkers, 1994 [SLLB96]*
    - ▷ *DEEP BLUE, chess, 1997: A historical event. [CHH02]*
    - ▷ *LOGISTELLO, Othello, 1997. [Bur97]*
- **2000's: the MCTS era**
  - **A "new" search technique based on Monte Carlo simulation (MCS) ($\sim$ 1993 AD) [BPW$^+$12].**
    - ▷ *Trying MCS based computer Go programs as early as 2004.*
    - ▷ *Beat alpha-beta based Go programs in 2007 after adding UCB and MCTS techniques.*
    - ▷ *Reach about 1 dan in the year 2010 and improve steadily until about 4 dan at 2012.*
    - ▷ *The program Zen beat a 9-dan professional master at March 17, 2012.*
      - *First game: five stone handicap and won by 11 points.*
      - *Second game: four stones handicap and won by 20 points.*
    - ▷ *Try to find applications in other games.*
    - ▷ *The improvement in performance has not been too much in recent years.*

History (3/5)

- **2012 until now: the AlphaGo era**
  - **Previous approach:**
    - *First to design a good evaluation function to approximately know goodness of a position.*
    - *Then, use a good search algorithm to search for a path leading to a position with the best possible score evaluated.*
    - *When the solution depth is huge and it is difficult to come up with a good evaluation function, then this approach works poorly.*
  - **Combining knowledge obtained from data mining and mostly machine learning with state of the art searching algorithms, Go has achieved the status of beating human top experts even on 19x19 boards.**
    - *Obtain a prediction for the set of plausible next moves.*
    - *Obtain a prediction of the final results.*
    - *Using these knowledge to aid the searching process.*
    - *Currently works better with Monte-Carlo (MCTS) based search engine, but this enhancement can be used with alpha-beta based searching as well.*

History (4/5)

- **AlphaGo (2016)**
  - Supervised learning
  - AlphaGo beat a top human player with the record of 4 vs 1 in March 2016 [SHM+16], and defeated the top human player with the record of 3 vs 0 in May 2017 [SSS+17].
    - ▷ *Supervised learning.*
    - ▷ *Another history was made.*
- **AlphaGo Zero (2017)**
  - Unsupervised learning.
    - ▷ *Use only rules, not expert knowledge*
  - Achieve the level of AlphaGo in days!

History (5/5)

- **AlphaZero (2018)**
  - **Unsupervised learning.**
    - *Works on games other than Go*
    - *Example: Chess Shogi*
  - **Claimed to be able to use on many other games and applications.**
- **MuZero (2020)**
  - **No use of rules**
  - **Know only the evaluation of the current situation, the set of possible actions (policy), and the consequence (reward)**
  - **Use on many board games and also on the video game Atari**

Definition of Games

- **A branch of the game theory that is called sequential games where each player takes turn making moves and can use information available in the previous moves.**
- **A game consists of**
  - **a set of feasible states**
    - ▷ *example: all legal arrangements of digits from 1 to 9 into a 9x9 matrix so that each row, column and 3x3 sub-matrix do not have same digits in Sudoku.*
  - **an initial state and a set of goal states**
    - ▷ *In Go, an initial empty board, and a terminal position where you have more territories.*
  - **A set of playing rules to move one state to another state**
    - ▷ *example: in 15-puzzle, slide a tile to an adjacent empty cell*
  - **A set of rules to decide whether the game is terminated or not.**
    - ▷ *King capturing*
    - ▷ *No legal next move*

Goals of games

- Categorize types of games based on the goal to achieve.
  - Opponent has no legal next move
    - *King-capturing: a side with no "king" cannot move such as Chess.*
    - *Elimination: making your opponent to have no pieces left to move such as Chinese dark chess.*
    - *Blocking: no way to move such as every possible move is blocked by another piece in Chinese Chess.*
  - Scoring: the one with more scores wins
    - *Territory: such as Go*
    - *Pieces count: such as Othello*
    - *Tokens: such as Monopoly*
  - Predefined terminal condition
    - *Pattern based: such as Connect-5*
    - *Other easily recognized condition: such as Hex or Sudoku*
  - Hybrid: a combination of the above such as EWN has a predefined location to reach, but also use the elimination rule.

Board games (1/2)

- **Classical board game** is a popular and history-rich branch of games **enjoyed** by human.
- A board of cells with an interconnected topology.
  - 4-connected for Go.
  - 6-connected for Hex.
  - Maybe a particular graph as in the game BattleShip.
- States
  - Initial state
  - Legal states
  - Goal states
    - ▷ *King-capturing: e.g. chess*
    - ▷ *Achieve the largest score: e.g. Go*
    - ▷ *Reach a particular arrangement: e.g. Sudoku*
- Playing rules
  - Define how a state can transit to another state
  - Can you pass or not
  - What to do when you have no legal next play
  - History dependent issues
    - ▷ *Loop handling such as Ko in Go.*

Board games (2/2)

- **Piece symmetry: whether all pieces are of the same**
  - *Go: all the same*
  - *chess: have different type of pieces*
- **Moving symmetry: whether all pieces can do the same types of moves**
  - *Go: all the same*
  - *chess: different type of pieces have different characteristics*
- **Capturing symmetry: whether a piece can capture any other piece**
  - *full symmetry: in EWN a piece can capture any other piece including your own.*
  - *partial symmetry: in Chinese chess, any piece can capture any opponent's piece*
  - *asymmetry: in Chinese dark chess: only a higher ranked one can capture a lower-ranked one with exceptions on Cannons.*
- **Piece promoting rules**

Taxonomy of games (1/3)

- **According to number of players**
  - Single player games: puzzles
  - Two-player games
  - Multi-player games
- **According to whether all players use the same set of rules.**
  - Symmetry: such as connect-5.
  - Asymmetry: such as Renju where the first and second players have different goals.
- **According to state information obtained by each player:**
  - Perfect-information games: all players have all the information they need to make a correct decision.
  - Imperfect-information games: some information is only available to selected players, e.g., you cannot see the opponent's cards in Poker.

Taxonomy of games (2/3)

- **According to number of players**
- **According to whether all players use the same set of rules.**
- **According to state information obtained by each player:**
- **According to rules of games known in advance:**
  - **Complete information games: the "rules" of the game are fully known by all players in advance.**
  - **Incomplete-information games: partial rules are not given in advance for some players.**
    - ▷ ***Example 1: As in the case of an "auction" where the goals and values of each bidder are unknown initially.***
    - ▷ ***Example 2: In a tournament, you do not know whether a player will try to draw or need to win a game due to the strategy he is using.***

Taxonomy of games (3/3)

- **According to number of players**
- **According to whether all players use the same set of rules.**
- **According to state information obtained by each player:**
- **According to rules of games known in advance:**
- **According to whether players can fully control the playing of the game:**
  - **Stochastic games: there is an element of chance such as dice rolls during playing.**
    - ▷ ***It is not clear whether the initialization phase can be counted as "during playing."***
    - ▷ ***For example: the initial dispatching of cards in Bridge after which is deterministic.***
  - **Deterministic games: the players have a full control over the games.**

Computational complexities of games

- **Single-player games are often called puzzles.**
  - **They have a single decision maker.**
  - **They are enjoyable to play.**
  - **A puzzle should have a solution which**
    - *is aesthetically pleasing;*
    - *gives the user satisfaction in reaching it.*
  - **Many puzzles require the solution to be unique.**
    - *NoNogram.*
    - *Sudoku.*
  - **Many puzzles are proven to be NP-complete.**
    - *24 puzzles including Light Up, Minesweeper, Solitaire and Tetris are NP-complete [G. Kendall et al. 2008].*
- **Many 2-player games are either PSPACE-complete or EXPTIME-complete.**
  - **Othello is PSPACE-complete, and Checkers and Chess are EXPTIME-complete [E.D. Demaine & R.A. Hearn 2001] [DH09].**
  - **Without Ko, Go is PSPACE-hard; with Japanese Ko, it is EXPTIME-complete; with the super-Ko rule, it is much harder.**

New frontiers

- **Traditional games: using paper and pencil, board, cards, and stones.**
- **Interactive computer games**
  - **Text-based interface during early days.**
  - **2-D graphics during the 1980's with the advance of personal computers.**
  - **3-D graphics with sound and special effects today.**
- **Human with the helps of computer software and hardware.**
- **On-line games: players compete against other humans or computer agents.**
- **Challenges:**
  - **Better user interface: such as Wii, AR, VR and holographic display.**
  - **Developing realistic characters.**
    - ▷ ***So far very primitive: simple rule-based systems and finite-state machines.***
    - ▷ ***Need researches in "human intelligence."***
  - **Educational purpose.**
- **Physical games played by machines: RoboCup.**

Concluding remarks (1/3)

- **A long universal history for human to play games.**
  - **A set of game pieces made 3000 BC! [Lorenzi 2013]**
  - **Not only human loves to play games, so are animals [Reinhold et al 2019]**
    - ▷ *Animals play games with human.*
    - ▷ *Animals play games themselves.*
  - **May be a primary condition of the generation of human cultures [Dalton and Luongo 2019]**
    - ▷ *voluntary*
    - ▷ *different from reality*
    - ▷ *internally, not material, motivated*
    - ▷ *some games are part of the learning through simulation, but not all of them seem to be so such as the sense of freedom*
    - ▷ *勤有功, 戲無益(?) — 三字經(∼ 1150 AD)*
  - **May be something essential for all life forms.**
    - ▷ *Modern biologist thinks the purpose of life forms is to survival, and if cannot, to reproduce so that survise through descendents*
    - ▷ *食色性也— 孟子告子篇 (∼ 300 BC)*
    - ▷ *Maybe they are not all of the human nature*

Concluding remarks (2/3)

- The starting of a avenue of research named "recreational mathematics" (娛悅數學).
  - Using games and puzzles for science education to the general population..
  - Sam Lloyd: Cyclopedia of Puzzles
  - Martin Gardener: monthly column in Scientific American
  - Berlekamp, Conway and Guy: Winning ways
- 雖小道，必有可觀者焉；致遠恐泥，是以君子不為也— 子夏 ($\sim$ 500 BC)
  - 六藝（周禮）：禮、樂、射、御、書、數。 ($\sim$ 450 BC)
  - 四藝（明末清初）：琴、棋、書、畫。 ($\sim$ 1600 AD)
- 寓教於樂
- Become one of the items used for school selection criteria.
- Interesting remark: animals also enjoy play games. Could this be something built-in in the evolution process?

Concluding remarks (3/3)

- **Arthur Samuel, 1960.**
  - Programming computers to play games is but one stage in the development of an understanding of the methods which must be employed for the machine simulation of intellectual behavior.
  - As we progress in this understanding it seems reasonable to assume that these newer techniques will be applied to real-life situations with increasing frequency, and the effort devoted to games ... will decrease.
  - Perhaps we have not yet reached this turning point, and we may still have much to learn from the study of games.

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