# Theory of Computer Games: Selected Advanced Topics

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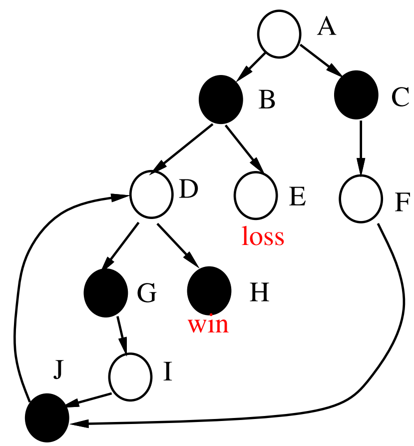
# Abstract

- **Some advanced research issues.**
  - **The graph history interaction (GHI) problem.**
  - **Opponent models.**
  - **Multi-player game tree search.**
  - **Bit board speedup.**
  - **Proof-number search.**
- **More research topics.**
  - **The influence of rules on games.**
    - *Allowing long cycles in Go.*
    - *The scoring of a suicide ply in chess.*
  - **Why a position is difficult to human?**
  - **Unique features in games.**

# Graph history interaction problem

- **The graph history interaction (GHI) problem [Campbell 1985]:**
  - **In a game graph, a position can be visited by more than one paths from a starting position.**
  - **The value of the position depends on the path visiting it.**
    - *It can be win, loss or draw for Chinese chess.*
    - *It can only be draw for Western chess and Chinese dark chess.*
    - *It can only be loss for Go.*
- **In the transposition table, you record the value of a position, but not the path leading to it.**
  - **Values computed from rules on repetition cannot be used later on.**
  - **It takes a huge amount of storage to store all the paths visiting it.**
- **This is a very difficult problem to be solved in real time [Wu et al '05] [Kishimoto and Müller '04].**

# GHI: when loop draws

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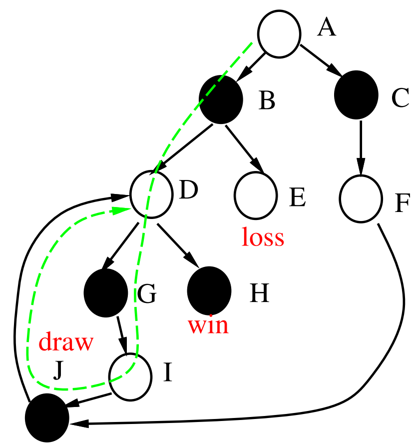


- **Assume if the game falls into a loop, then it is a draw.**
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  - ▷ ***Memorized $J$ as a draw position.***

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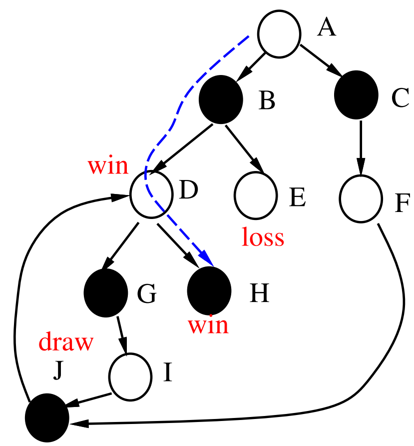


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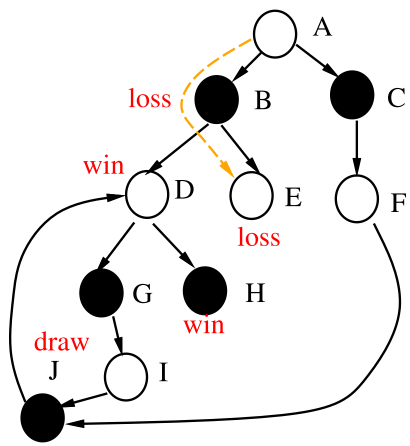


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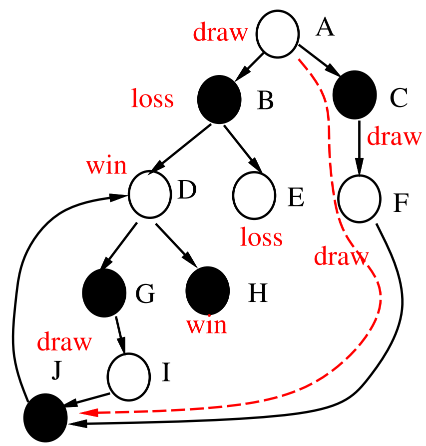


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- $A \rightarrow B \rightarrow D \rightarrow H$ **is a win. Hence $D$ is win.**
- $A \rightarrow B \rightarrow E$ **is a loss. Hence $B$ is loss.**
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- $A$ **is draw because one child is loss and the other chile is draw.**

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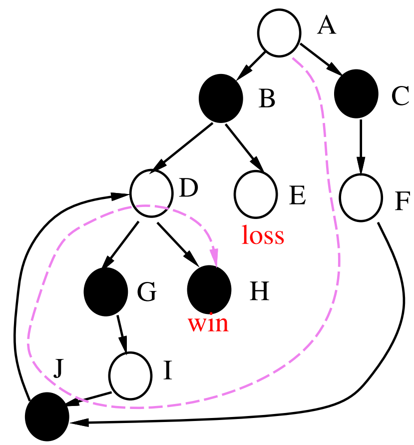


- Assume if the game falls into a loop, then it is a draw.
- $A \rightarrow B \rightarrow D \rightarrow G \rightarrow I \rightarrow J \rightarrow D$ is draw by rules of repetition.
  - ▷ *Memorized $J$ as a draw position.*
- $A \rightarrow B \rightarrow D \rightarrow H$ is a win. Hence $D$ is win.
- $A \rightarrow B \rightarrow E$ is a loss. Hence $B$ is loss.
- $A \rightarrow C \rightarrow F \rightarrow J$ is draw because $J$ is recorded as draw.
- $A$ is draw because one child is loss and the other chile is draw.
- However, $A \rightarrow C \rightarrow F \rightarrow J \rightarrow D \rightarrow H$ is a win (for the root).

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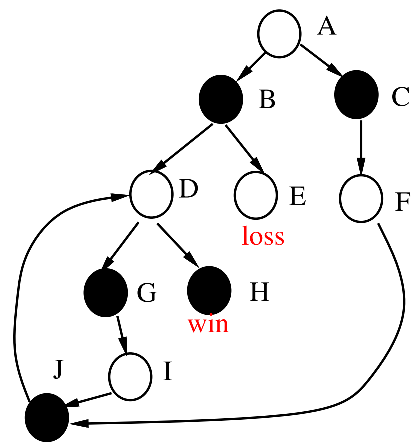


- **Assume the one causes loops wins the game.**

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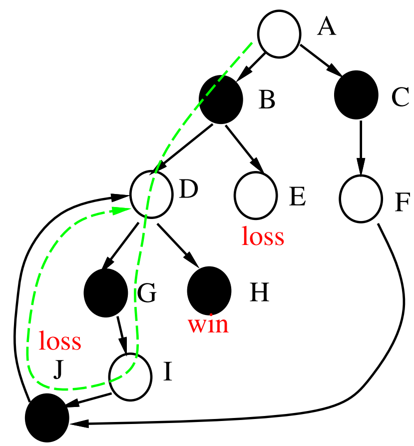


- **Assume the one causes loops wins the game.**
- $A \to B \to D \to G \to I \to J \to D$ **is loss because of rules of repetition.**
  - ▷ ***Memorized $J$ as a loss position (for the root).***

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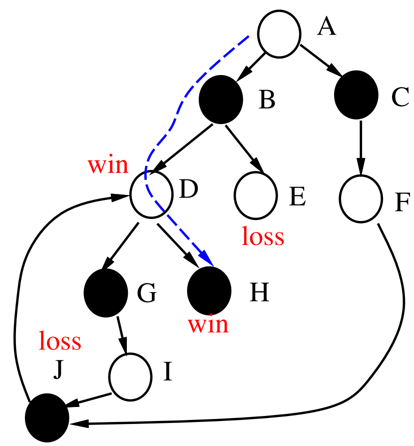


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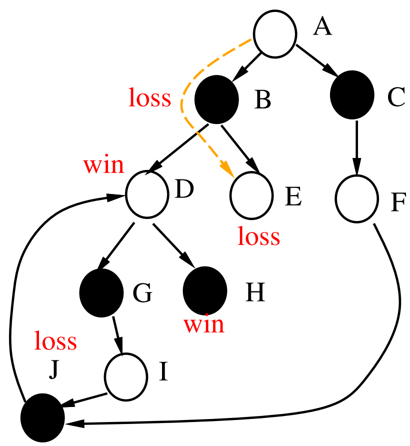


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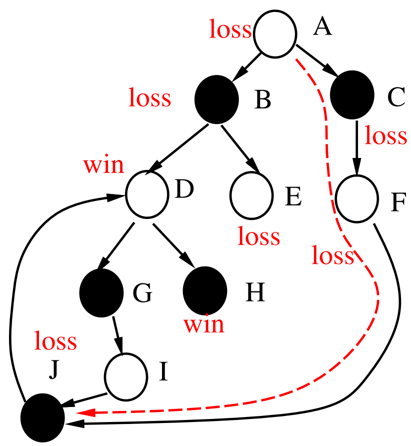


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- $A \rightarrow B \rightarrow D \rightarrow H$ **is a win. Hence** $D$ **is win.**
- $A \rightarrow B \rightarrow E$ **is a loss. Hence** $B$ **is loss.**
- $A \rightarrow C \rightarrow F \rightarrow J$ **is loss because** $J$ **is recorded as loss.**
- $A$ **is loss because both branches lead to loss.**

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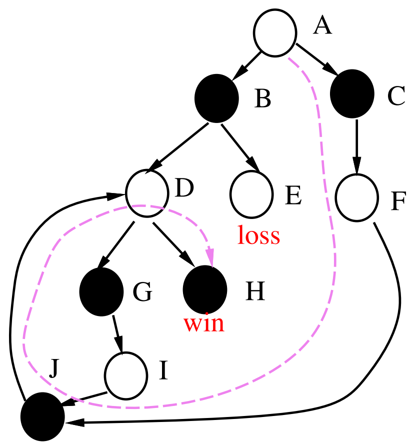



- **Assume the one causes loops wins the game.**
- $A \to B \to D \to G \to I \to J \to D$ **is loss because of rules of repetition.**
  - ▷ ***Memorized $J$ as a loss position (for the root).***
- $A \to B \to D \to H$ **is a win. Hence $D$ is win.**
- $A \to B \to E$ **is a loss. Hence $B$ is loss.**
- $A \to C \to F \to J$ **is loss because $J$ is recorded as loss.**
- $A$ **is loss because both branches lead to loss.**
- **However,** $A \to C \to F \to J \to D \to H$ **is a win (for the root).**

# Comments

- Using DFS to search the above game graph from left first or from right first produces two different results.
- Position $A$ is actually a win position.
  - Problem: memorize $J$ being draw is only valid when the path leading to it causes a loop.
- Storing the path leading to a position in a transposition table requires too much memory.
  - Maybe we can store some forms of hash code to verify it.
- Finding a better data structure for solving this problem remains to be a challenging research issue.
- Remark: It real settings, it is usually the case that the rule of loops is enforced after 3 repetitions. However, GHI problem holds for any times of repetition.

# Opponent models

- **In a normal alpha-beta search, it is assumed that you and the opponent use the same strategy.**
  - **What is good to you is bad to the opponent and vice versa!**
  - **Hence we can reduce a minimax search to a NegaMax search.**
  - **This is normally true when the game ends, but may not be true in the middle of the game.**
- **What will happen when there are two strategies or evaluation functions $f_1$ and $f_2$ so that**
  - **for some positions $p$, $f_1(p)$ is better than $f_2(p)$**
    - ▷ ***"better" means closer to the real value $f(p)$***
  - **for some positions $q$, $f_2(q)$ is better than $f_1(q)$**
- **If you are using $f_1$ and you know your opponent is using $f_2$, what can be done to take advantage of this information.**
  - **This is called OM (opponent model) search [Carmel and Markovitch 1996].**
    - ▷ ***In a MAX node, use $f_1$.***
    - ▷ ***In a MIN node, use $f_2$.***

# Other usage of the opponent model

- **Depend on strength of your opponent, decide whether to force an easy draw or not.**
  - **This is called the contempt factor.**
- **Example in CDC:**
  - **It is easy to chase the king of your opponent using your pawn.**
  - **Drawing a weaker opponent is a waste.**
  - **Drawing a stronger opponent is a gain.**
- **It is feasible to use a learning model to "guess" the level of your opponent as the game goes and then adapt to its model in CDC [Chang et al 2021].**

# Opponent models – comments

- **Comments:**
  - **Need to know your opponent's model precisely or to have some knowledge about your opponent.**
  - **How to learn the opponent model on-line or off-line?**
  - **When there are more than 2 possible opponent strategies, use a probability model (PrOM search) to form a strategy.**
- **Remark: A common misconception is that if your opponent uses a worse strategy $f_3$ than the one, namely $f_2$, used in your model, then he may get advantage.**
  - **This is impossible if $f_2$ is truly better than $f_3$.**
  - **If $f_1$ can beat $f_2$, then $f_1$ can sure beat $f_3$.**

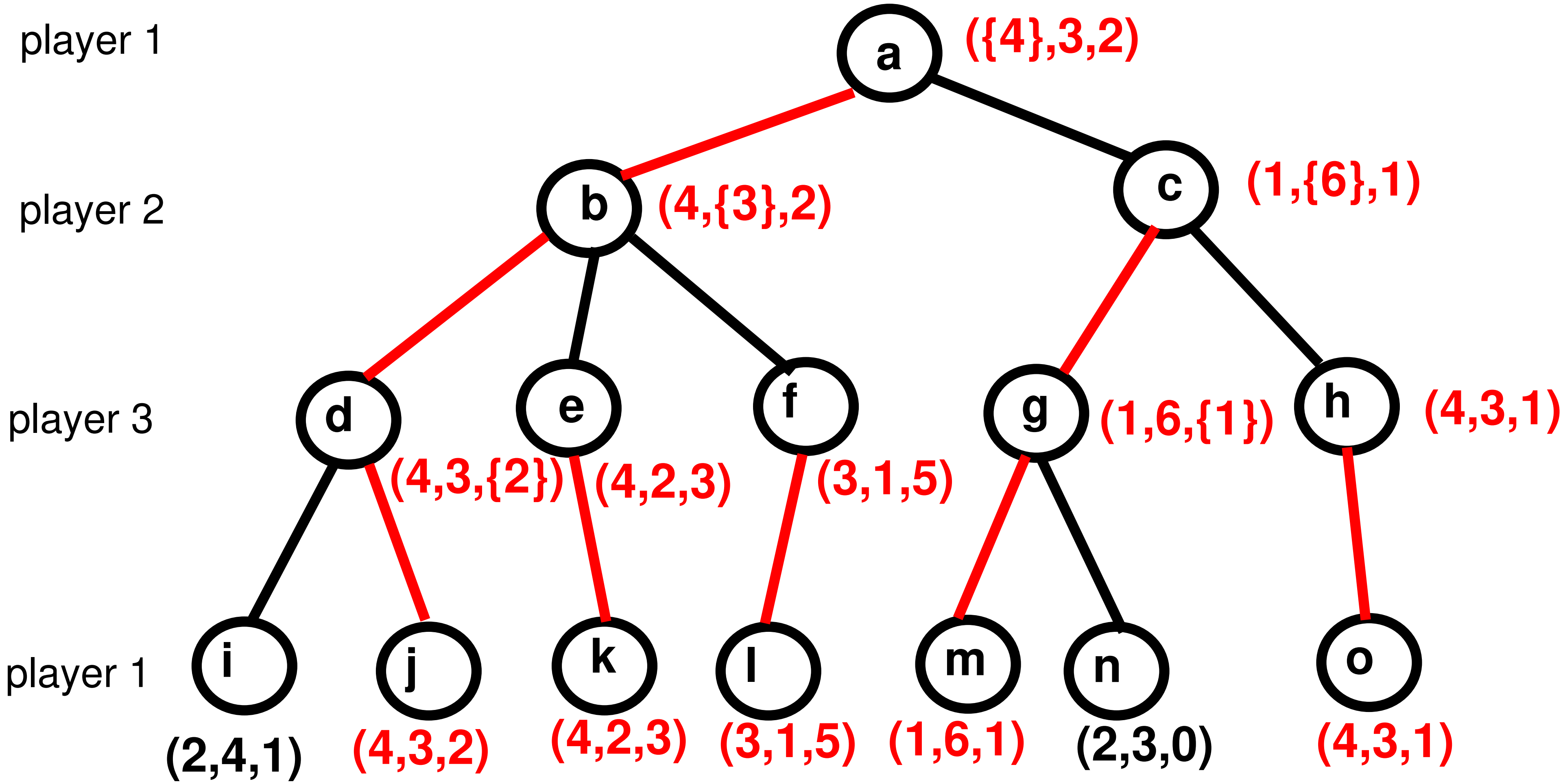
# Multi-player game tree search

- **Games with more than 2 players.**
  - **Mahjong: 4 players**
  - **Contract bridge or bridge: 4 players**
  - **Monopoly: 2 to many players**
  - **Scrabble: 2 to 4 players**
  - **Risk: 2 to 6 players**
- **Assume we have $n$ players, $y_1, \ldots, y_n$ in a game.**
  - **We have $n$ evaluating functions, $score_i$, one for each player.**
  - **Given a position $p$ with the children $p_1, \ldots, p_m$, let $score_i(p)$ be the score of $y_i$ for $p$.**
    - ▷ ***If $p$ is a terminal position for $y_i$, then $m = 0$ and $score_i(p)$ is the "true" score of $y_i$ in $p$.***
    - ▷ ***Otherwise, $score_i(p) = \max_{j=1}^{m} score_i(p_j)$.***
  - **The above algorithm is called MAX$^n$ where stands for during each turn, each player maximizes his own score without considering scores of others.**

# MAX$^n$: algorithm

- $next\_player(idx)$: **the player who is next to player** $idx$.
- **Brute force algorithm for multi-player games.**
- **Algorithm MAXN(position** $p$**, player** $idx$**)**
  - **output:** $best$ **which is an array with** $best[i]$ **being the best value for player** $i$ **so far.**
  - **If** $p$ **is terminal, then return** $best[i] = score_i(p), \forall i$;
  - **initialize** $best$ **to be** $best[i] = -\infty, \forall i$;
  - **Let** $p_i$ **be the** $i$**th child of** $p$;
  - **for** $i = 1$ **to last child of** $p$ **do**
    - ▷ $current$ = ***MAXN(***$p_i$***, next_player(***$idx$***));***
    - ▷ ***if*** $current[idx] > best[idx]$, $best = current$; ***// maximized player idx***
  - **return** $best$;

# MAX$^n$: example ($n = 3$)



# Opportunities for pruning (1/2)

- Let $p$ be a position in a multi-player game.
- Alpha-beta pruning is a special case for $n = 2$ and cannot be generalized for $n > 2$.
  - Property used in alpha-beta pruning:
    - ▷ ***What is good for $y_1$ is definitely bad for $y_2$ by using the zero sum principle which is for a position $p$, $score_1(p) + score_2(p) = 0$.***
  - The above may not be true for $n > 2$.
    - ▷ ***When $n = 3$, what is good for $y_1$ may be also good for $y_2$, but very bad for $y_3$.***

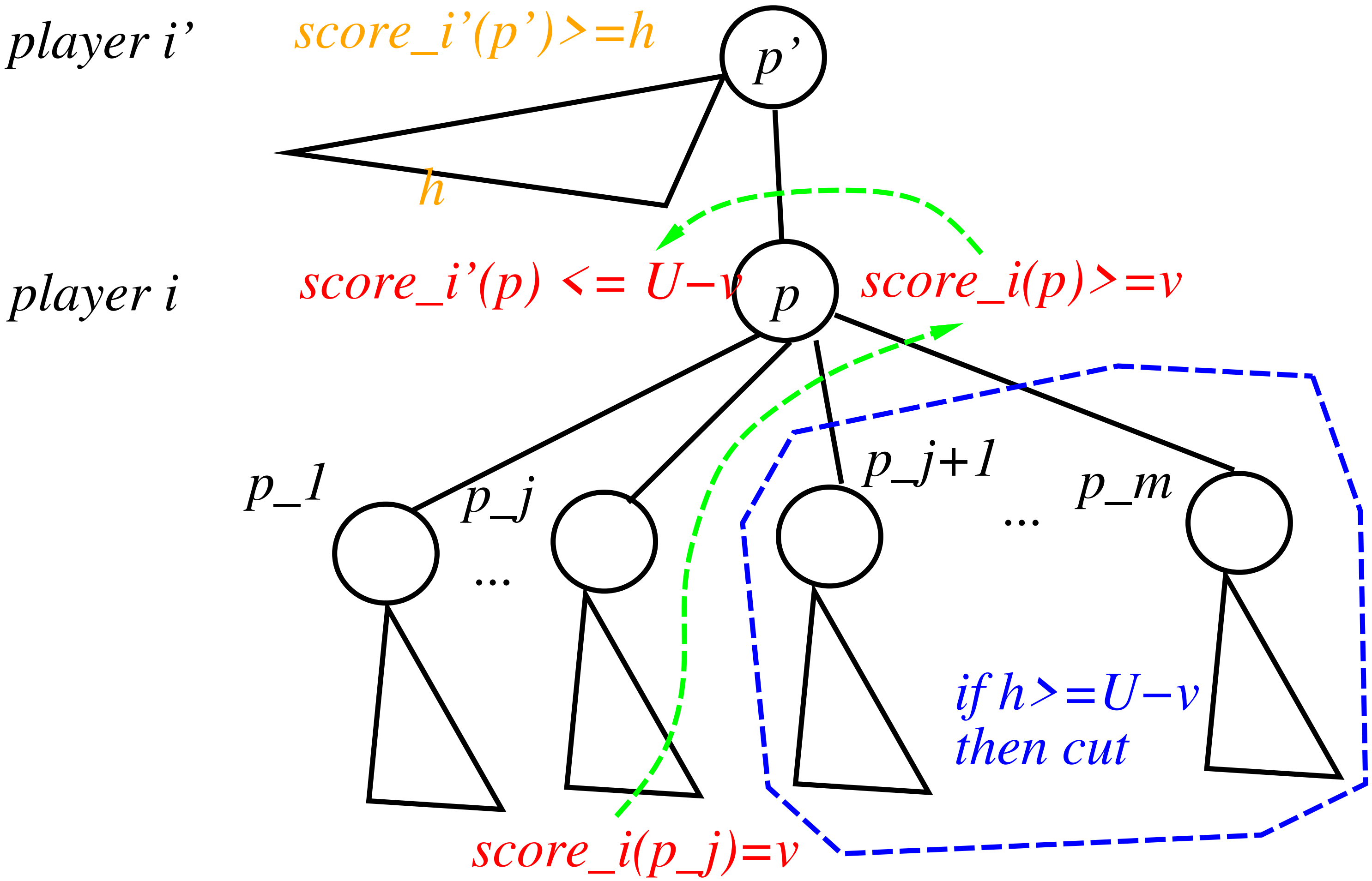
# Opportunities for pruning (2/2)

- **For a position $p$, if there is no constraints on the $n$ scores of $p$, then it is impossible to have any cut offs for MAX$^n$.**
  - **In applications we often have the following properties.**
    - ▷ ***Zero sum.***
    - ▷ ***The sum of all $n$ scores for $p$ has an upper bound $U$.***
    - ▷ ***The score of $p$ for any player has a lower bound $L$.***
  - **Examples:**
    - ▷ ***Go for $n$ players: each player owns pieces of a distinct color.***
      ***$\rightarrow$ the sum of all points $\leq$ the board size, and the score cannot be negative.***
    - ▷ ***Othello for $n$ players: each player owns pieces of a distinct color and flips all pieces of different colors.***
      ***$\rightarrow$ the sum of all points $\leq$ the plys played so far and the score cannot be negative.***

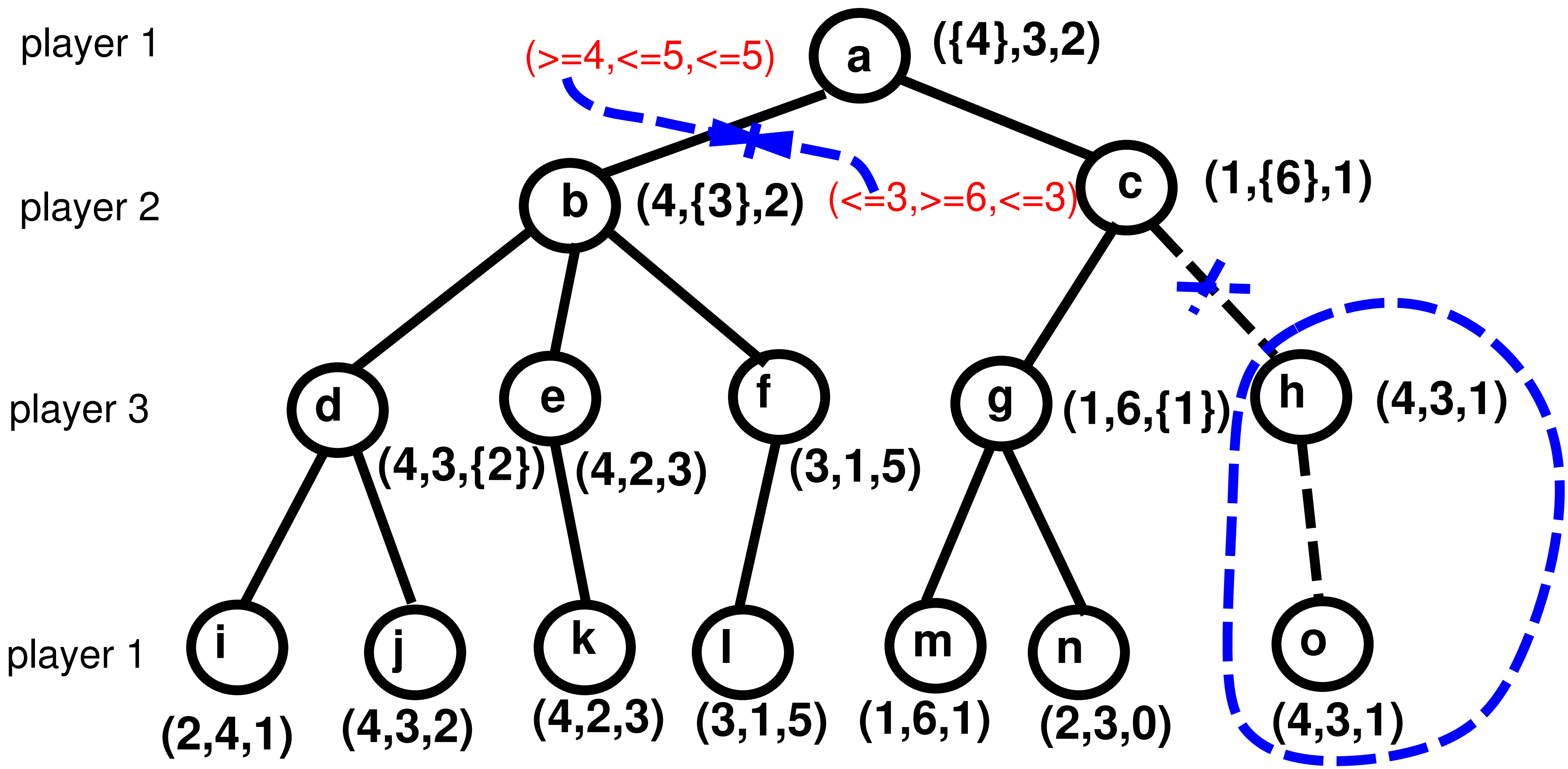
# Pruning

- **Recall: a position $p$ with the children $p_1, \ldots, p_m$ and the parent $p'$, and $score_i(p)$ is the score of player $i$ for $p$.**
- **Direct pruning:**
  - **During the turn of the $i$th player, if $score_i(p_j) = U$, then no more search is needed.**
- **Shallow pruning:**
  - **Without loss of generality, assume $L = 0$.**
  - **During the turn of the $i$th player, if $score_i(p_j) = v$ so far, then $score_i(p) \geq v$ since each player is a max player.**
  - **This implies $score_j(p) \leq U - v$ if $j \neq i$.**
  - **Let $i'$ be the index of the immediate previous player.**
  - **We know $score_{i'}(p') \geq h$ if he has done some searching.**
  - **If $h \geq U - v$, then we have a cut off.**

# MAX$^n$: ideas for cutoff



# MAX$^n$: cutoff example ($n = 3, U = 9$)



# Remarks about pruning in MAX$^n$

- **Direct pruning is a degenerated case of the shallow pruning by the following settings.**
  - **If $v = U$, then the scores of all other players are all zero.**
  - **Using the lower bound $L$, you can get a cut off.**
- **Compared to two-player alpha-beta pruning, both direct and shallow pruning can be used in $n \geq 2$.**
- **Deep pruning does not work when $n > 2$.**
  - **Assume you are searching the node $w$, $v$ is your parent and $u$ is an ancestor that is not $v$.**
  - **Assume node $x$ is the turn of player $player(x)$.**
  - **Any value of $score_{player(u)}(u)$ cannot produce any cutoff on searching the tree $T_w$ because $player(v)$ makes the decision first in propagating the values up.**
  - **Any value of $score_{player(u)}(w)$ can be propagated up and be used by $u$.**

# Algorithm for shallow cut off

- **Functions and data structures**
  - $next\_player(idx)$: **the player who is next to player** $idx$.
  - $score_i(p)$: **the score of player** $i$ **for the position** $p$.
  - $U$: **the upper bound of sum of all scores among all players on a position.**
  - **Assume** $L$ **is** $0$.
  - $best$ **and** $current$ **are both arrays of size** $n$.
- **Algorithm shallow(position $p$, player $idx$, value $bound$)**
  - **return value:** $best$ **which is an array with** $best[i]$ **being the best value for player** $i$ **so far.**
  - **If** $p$ **is terminal, then return** $best[i] = score_i(p), \forall i$;
  - **Let** $p_i$ **be the** $i$**th child of** $p$;
  - $best = shallow(p_1, next(idx), U)$; **// recursive call on the first child**
  - **for** $i = 2$ **to last child of** $p$ **do**

    *4.1:* **if** $best[idx] = U$, **then return** $best$ **// immediate cut off**
    *4.2:* **if** $best[idx] \geq bound$, **then return** $best$ **// shallow cut off**
    *4.3:* $current = shallow(p_i, next\_player(idx), U - best[idx])$;
    *4.4:* **if** $current[idx] > best[idx]$, $best = current$; **// maximize player idx**

  - **return** $best$;

# Comments

- **A generalization of alpha-beta cutoff on adjacent depths.**
- **Does not work on deep alpha-beta cutoff [Korf 1991].**
- **In the best case, the effective branching factor is $\frac{1+\sqrt{4b-3}}{2}$ where $b$ is the average branching factor.**
  - **Comparing to alpha-beta cut off, the best effective branching factor is $\sqrt{b}$.**
- **In the average case, the effective branching factor is approaching $O(b)$.**
  - **Comparing to alpha-beta cut off, the the average effective branching factor is $b^{0.75}$ [Fuller et al 1975].**
  - **This implies most of the cut off come from deep pruning in the average case.**
- **More research are needed to get more cutoff by observing additional constraints on the values from the application domain.**
- **MCTS can be easily extended to work on any number of players, but need to work on better properties of convergence.**

# Hardware Speedup

- **Using hardware to speed up searching is not new.**
  - **Parallel computing.**
    - ▷ *The Northwestern University CHESS program series on the 1970's makes full usage of hardware advantages from supercomputers [Atkin & Slate 1977].*
  - **Special hardware acceleration:**
    - ▷ *Belle: a chess machine with special micro instructions for move generation, alpha-beta pruning and transposition table operations [Condon & Thompson 1982].*
    - ▷ *Deep Blue: custom VLSI FPGA chips for operating chess playing expert systems [Hsu et al 1995].*
- **The above's are very costly.**

# Bit board techniques

- **Everyone can make use of the benefits of hardware acceleration now by smart usage of fast parallel bitwise operations provided by modern day CPU's.**
  - **Intel CPU's: MMX and SSE [Intel 2021]**
  - **AMD: 3D Now! [AMD 2000]**
- **Main technique**
  - **Using bits to represent the board and pieces on the board.**
    - *Transfer a board into an $n \times m$ picture*
    - *Transfer pieces into patterns of pixel rectangles*
  - **These instructions are usually in the form of SIMD (single instruction multiple data).**
  - **Many are for image related operations.**
  - **May also make use of GPU.**

# Special instruction sets (1/2)

- **Make use of fast parallel bitwise operations provided by modern day CPU's.**
- **Many different types**
  - **Find aggregated information**
  - **Parallel bit deposit and extract**
  - **...**
- **Most of the instructions can be done using AND, OR, NOT operations, but can be done much faster using special CPU instructions.**

# Special instruction sets (2/2)

- **Find aggregated information:**
  - **population count (POPCNT): the number of 1-bits in a "word".**
  - **leading/trailing zero count: LZCNT, TZCNT**
- **Parallel bit deposit and extract**
  - **Pack in sequence selected bits (PEXT): extract something out**
    - ▷ $PEXT(W, Mask)$ ***returns a word by packing to the right those bits in the word*** $W$ ***whose corresponding bits in the word*** $Mask$ ***are equal to*** $1$.
    - ▷ ***Example:*** $PEXT(010110010, 010101010)$ ***extracts the four even numbered bit and then pack it to the right. Thus it returns*** $01100$.
  - **Distribute bits in sequence to selected locations (PDEP): deposit something into.**
    - ▷ $PDEP(W, Mask)$ ***returns a word by sending the*** $i$***th bit in the word*** $W$ ***to the location addressed by the*** $i$***th*** $1$.
    - ▷ ***Example:*** $PEXT(01100, 010101010)$ ***deposits the four bits to the even numbered location. Thus it returns*** $010100000$.

# Example I

- **In Go, how to find the number of empty intersections on the board?**
  - **Assume you have a long hardware word $W$ of 19*2=38 bits.**
    - ▷ *Use 19 words $W_1, \ldots, W_{19}$ to represent the rows.*
  - **Encoding: bits $i$ and $i+1$ in $W_j$ represents the status of the intersection at the $i$th column and $j$th row.**
    - ▷ $00$ *means empty.*
    - ▷ $10$ *means a black stone.*
    - ▷ $01$ *means a white stone.*
  - **POPCOUNT($W_j$) gives the number of stones in the $j$th row.**
  - **$19-$POPCOUNT($W_j$) gives the number of empty intersections in the $j$th row.**

# Example II

- **In Chinese Dark Chess (CDC), how to find all revealed pieces of a color on the board?**
  - **Assume you have a long hardware word $W$ of 32*3=96 bits.**
  - **Encoding: bits $3i$, $3i+1$, and $3i+2$ in $W_b$ represents the status of the $i$th cell on the board with regard to the black side. Similarly, we have $W_r$ for the red side.**
    - ▷ $000$ *means empty, or pieces of other color or dark.*
    - ▷ $xyz$ *means the $xyz$th kind of piece where there are up to only 7 different kinds of pieces of a color. Thus the encodings used are from 1 to 7.*
- **Algorithm Find_PCES(color $c$)**
  - **// find all pieces of color $c$ and put them in m[]**
  - $i = 0$
  - **while $W_c \mathrel{!}= \ 0$ do**
    - ▷ $a = TZCNT(W_c)$ *// count the number of tailing zeros*
    - ▷ $a = a - a \mod 3$ *// find piece location*
    - ▷ $W_c >>= a$ *// right shift $a$ bits, find next piece*
    - ▷ $m[i++] = W_c \ \& \ 07$ *// gives a piece of color $c$*
    - ▷ $W_c \ \&= \sim(07)$ *// mask off the lowest 3 bits*
  - **return $m$**

# Example III

- **In Othello, how to pack information of a column in a continuous sequence of cells?**
  - **Problem:**
    - ▷ *The board of Othello is a 8 by 8 rectangle. Assume we use a word to represent the board and use the row-major ordering, then cells in a column are non-adjacent.*
    - ▷ *Example: The first (leftmost) column are numbered 0, 8, 16, 24, 32, 40, 48, and 56 in a row-major ordering.*
  - **Encoding:**
    - ▷ *Assume you have a hardware word $W$ of 64 bits.*
    - ▷ $W_b$ *and* $W_w$ *are words for black and white stones respectively.*
    - ▷ $0$ *means empty or other color.*
    - ▷ $(W_b|W_w)$ *gives the word for empty spaces.*
- **Algorithm Find_Column(color $c$, int $idx$)**
  - // pack information in column idx into adjacent bits
  - // $Loc$ is an array which gives the masks of bits in column idx
  - $Mask = Loc[idx]$
  - $W = PEXT(W_c, Mask)$
  - return $W$

# Comments

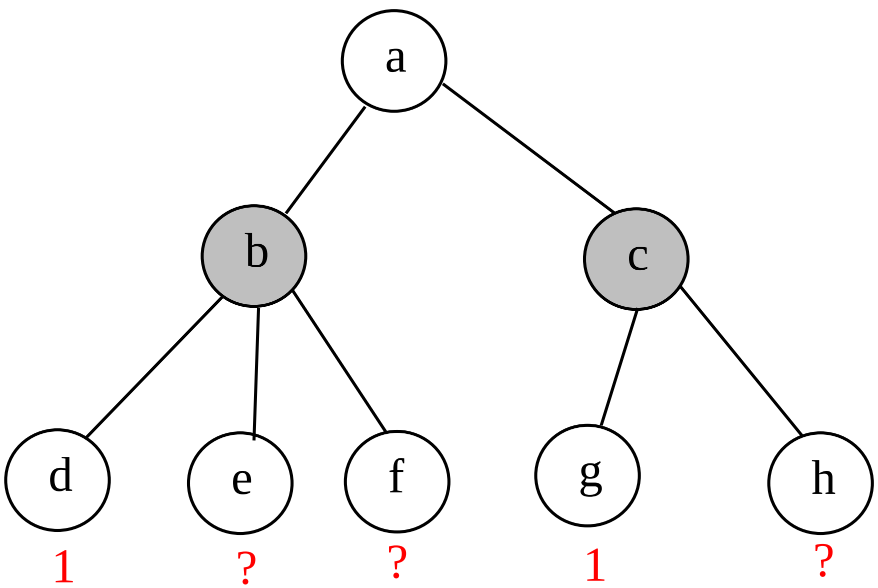
- Read carefully the instruction set of the CPU used to find out any special SIMD operations that are or aren't provided.
- The speedup is a lot, sometimes more than 50 times, if the encoding used is good [Browne 2014].

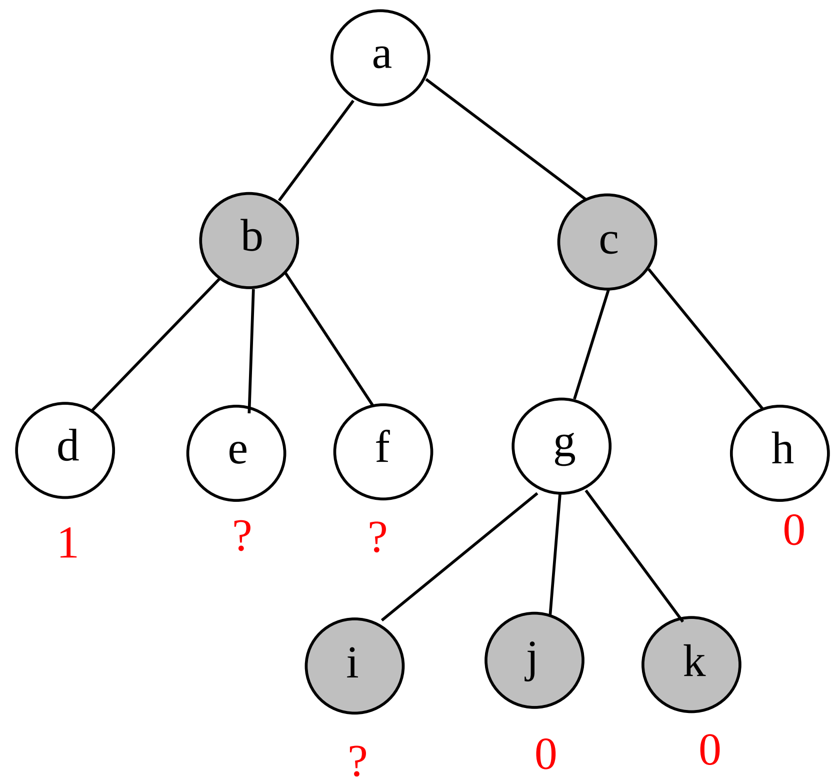
# Proof number search

- Consider the case of a 2-player game tree with either 0 or 1 on the leaves.
  - win, or not win which is lose or draw;
  - lose, or not lose which is win or draw;
  - Call this a binary valued game tree.
- If the game tree is known as well as the values of some leaves are known, can you make use of this information to search this game tree faster?
  - The value of the root is either 0 or 1.
  - If a branch of the root returns 1, then we know for sure the value of the root is 1.
  - The value of the root is 0 only when all branches of the root returns 0.
  - An AND-OR game tree search.

# Which node to search next?

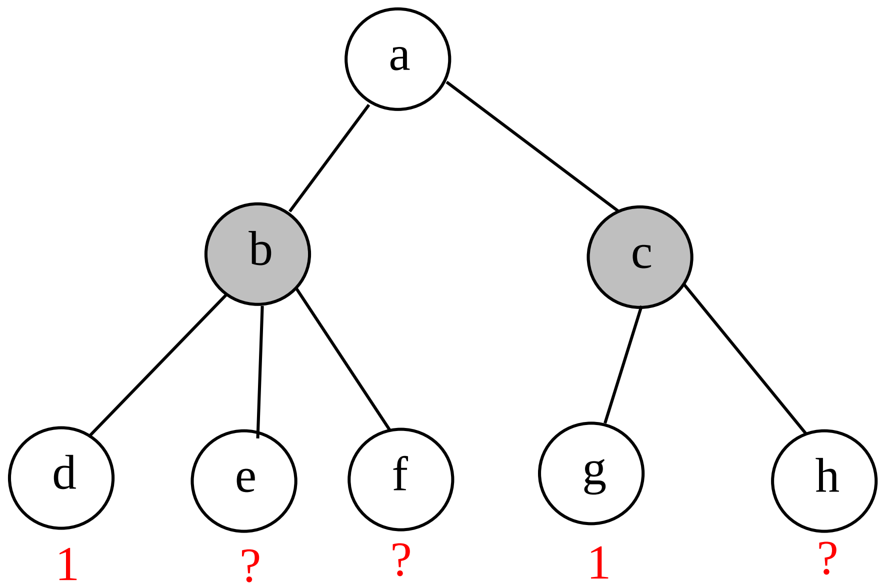
- A **most proving node** for a node $u$: a descendent node if its value is 1, then the value of $u$ is 1.
- A **most disproving node** for a node $u$: a descendent node if its value is 0, then the value of $u$ is 0.



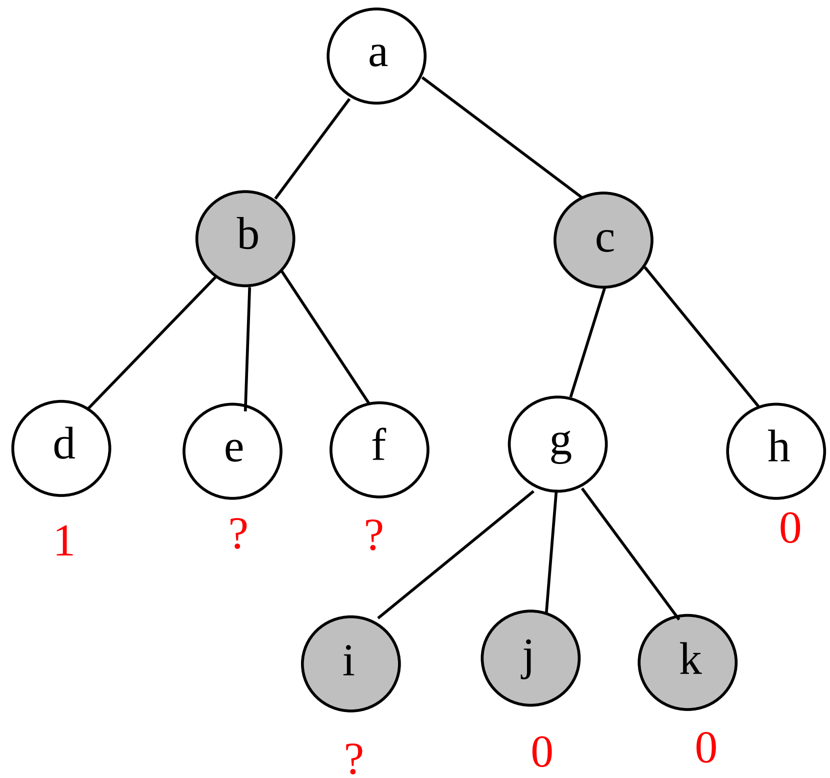
# Most proving node

- **Node $h$ is a most proving node for $a$.**



# Most disproving node

- **Node $e$ or $f$ is a most disproving node for $a$.**

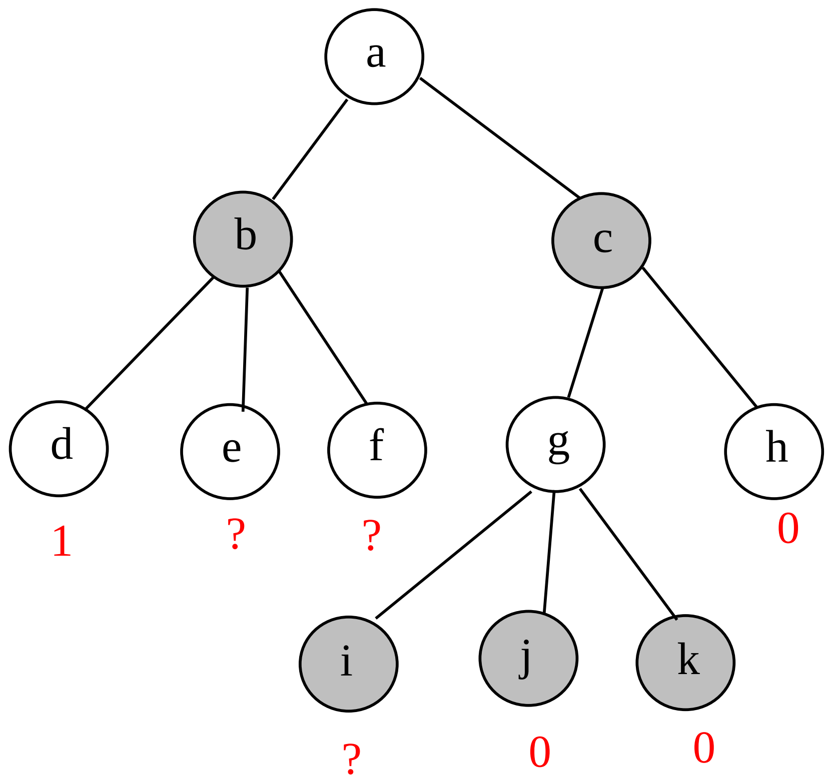


# Proof or Disproof Number

- **Assign a proof number and a disproof number to each node $u$ in a binary valued game tree.**
  - $proof(u)$**: the minimum number of leaves needed to visited in order for the value of $u$ to be 1.**
  - $disproof(u)$**: the minimum number of leaves needed to visited in order for the value of $u$ to be 0.**
- **The definition implies a bottom-up ordering.**

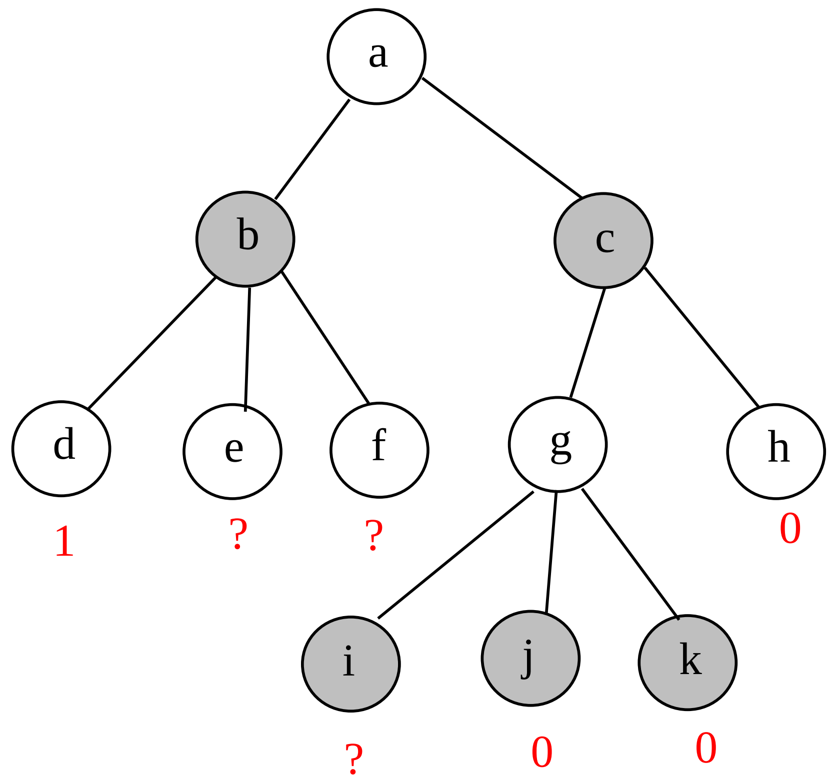
# Proof number

- **Proof number for the root $a$ is 2.**
  - ▷ ***Need to at least prove $e$ and $f$.***



# Disproof number

- **Disproof number for the root $a$ is 2.**
  - ▷ ***Need to at least disprove $i$, and either $e$ or $f$.***



# Proof Number: Definition

- $u$ **is a leaf:**
  - **If** $value(u)$ **is unknown, then** $proof(u)$ **is the cost of evaluating** $u$**.**
  - **If** $value(u)$ **is 1, then** $proof(u) = 0$**.**
  - **If** $value(u)$ **is 0, then** $proof(u) = \infty$**.**
- $u$ **is an internal node with all of the children** $u_1, \ldots, u_b$**:**
  - **if** $u$ **is a MAX node,**

$$proof(u) = \min_{i=1}^{i=b} proof(u_i);$$

  - **if** $u$ **is a MIN node,**

$$proof(u) = \sum_{i=1}^{i=b} proof(u_i).$$

# Disproof Number: Definition

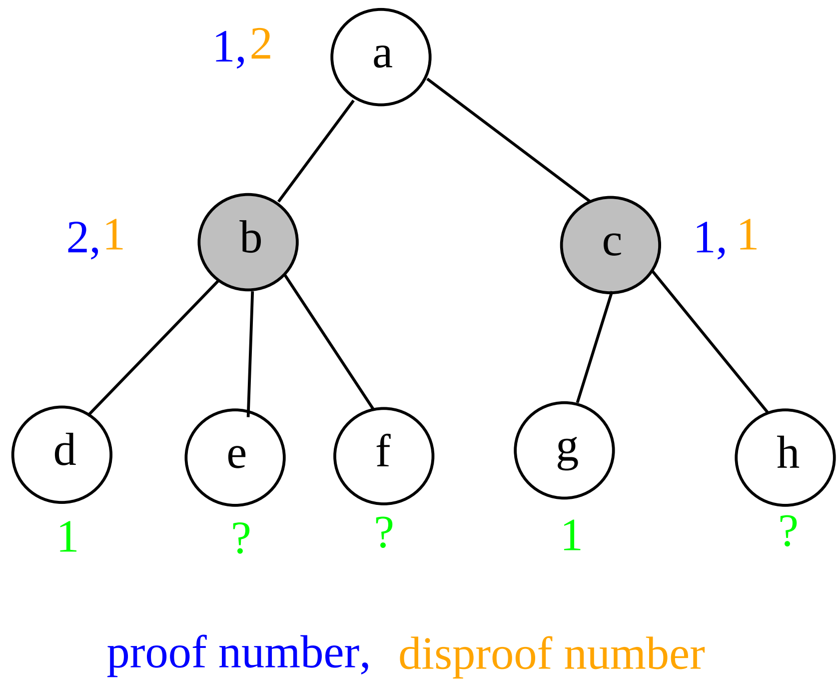
- $u$ **is a leaf:**
  - **If** $value(u)$ **is unknown, then** $disproof(u)$ **is cost of evaluating** $u$**.**
  - **If** $value(u)$ **is 1, then** $disproof(u) = \infty$**.**
  - **If** $value(u)$ **is 0, then** $disproof(u) = 0$**.**
- $u$ **is an internal node with all of the children** $u_1, \ldots, u_b$**:**
  - **if** $u$ **is a MAX node,**

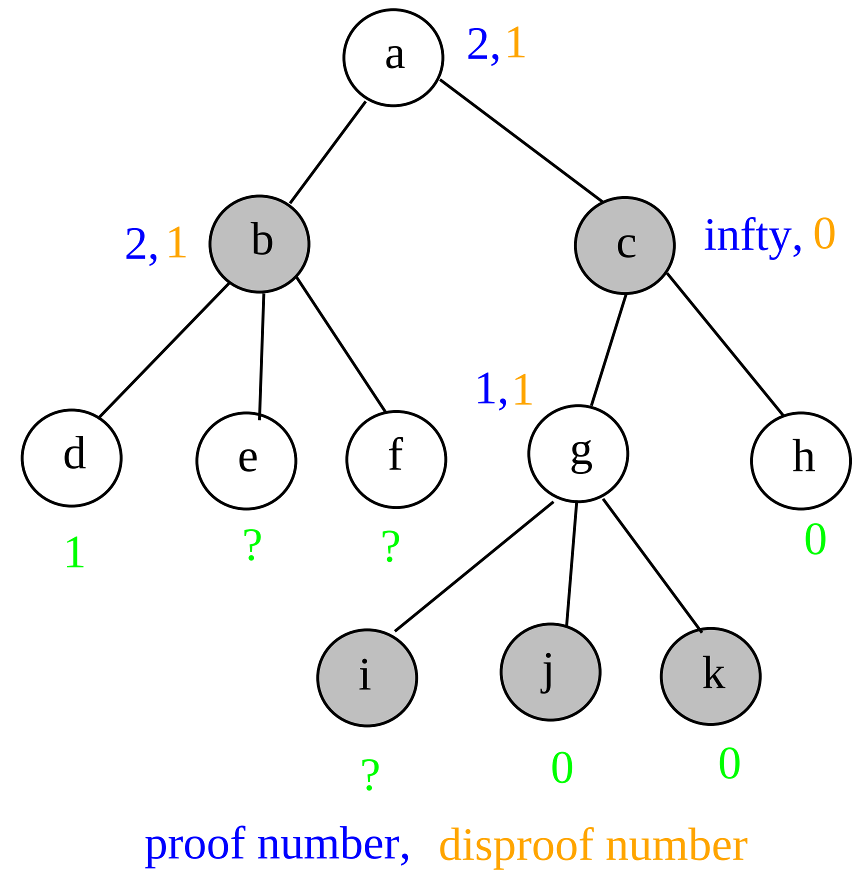
$$disproof(u) = \sum_{i=1}^{i=b} disproof(u_i);$$

  - **if** $u$ **is a MIN node,**

$$disproof(u) = \min_{i=1}^{i=b} disproof(u_i).$$

# Illustrations



# How these numbers are used (1/2)

- **Scenario:**
  - **For example, the tree $T$ represents an open game tree or an endgame tree.**
    - ▷ *If $T$ is an open game tree, then maybe it is asked to prove or disprove a certain open game is win.*
    - ▷ *If $T$ is an endgame tree, then maybe it is asked to prove or disprove a certain endgame is win o loss.*
    - ▷ *Each leaf takes a lot of time to evaluate.*
    - ▷ *We need to prove or disprove the tree using as few time as possible.*
  - **Depend on the results we have so far, pick a leaf to prove or disprove.**
- **Goal: solve as few leaves as possible so that in the resulting tree, either $proof(root)$ or $disproof(root)$ becomes $0$.**
  - **If $proof(root) = 0$, then the tree is proved.**
  - **If $disproof(root) = 0$, then the tree is disproved.**
- **Need to be able to update these numbers on the fly.**

# How these numbers are used (2/2)

- **Let $GV = \min\{proof(root), disproof(root)\}$.**
  - **$GT$ is "prove" if $GV = proof(root)$, which means we try to prove it.**
  - **$GT$ is "disprove" if $GV = disproof(root)$, which means we try to disprove it.**
  - **In the case of $proof(root) = disproof(root)$, we set $GT$ to "prove" for convenience.**
- **From the root, we search for a leaf whose value is unknown.**
  - **The leaf found is a most proving node if $GT$ is "prove", or a most disproving node if $GT$ is "disprove".**
  - **To find such a leaf, we start from the root downwards recursively as follows.**
    - ▷ ***If we have reached a leaf, then stop.***
    - ▷ ***If $GT$ is "prove", then pick***
      ***a child with the least proof number for a MAX node, and***
      ***any node that has a chance to be proved for a MIN node.***
    - ▷ ***If $GT$ is "disprove", then pick***
      ***a child with the least disproof number for a MIN node, and***
      ***any node that has a chance to be disproved for a MAX node.***

# PN-search: algorithm (1/2)

- **{$*$ Compute and update proof and disproof numbers of the root in a bottom up fashion until it is proved or disproved. $*$}**
- *loop:*
  - **If** $proof(root) = 0$ **or** $disproof(root) = 0$**, then we are done, otherwise**
    - ▷ $proof(root) \leq disproof(root)$***: we try to prove it.***
    - ▷ $proof(root) > disproof(root)$***: we try to disprove it.***
  - $u \leftarrow root$; **{$*$ find a leaf to prove or disprove $*$}**
  - **if we try to prove, then**
    - ▷ ***while*** $u$ ***is not a leaf do***
    - ▷ ***if*** $u$ ***is a MAX node, then***
      $u \leftarrow$ ***leftmost child of*** $u$ ***with the smallest non-zero proof number;***
    - ▷ ***else if*** $u$ ***is a MIN node, then***
      $u \leftarrow$ ***leftmost child of*** $u$ ***with a non-zero proof number;***
  - **else if we try to disprove, then**
    - ▷ ***while*** $u$ ***is not a leaf do***
    - ▷ ***if*** $u$ ***is a MAX node, then***
      $u \leftarrow$ ***leftmost child of*** $u$ ***with a non-zero disproof number;***
    - ▷ ***else if*** $u$ ***is a MIN node, then***
      $u \leftarrow$ ***leftmost child of*** $u$ ***with the smallest non-zero disproof number;***

# PN-search: algorithm (2/2)

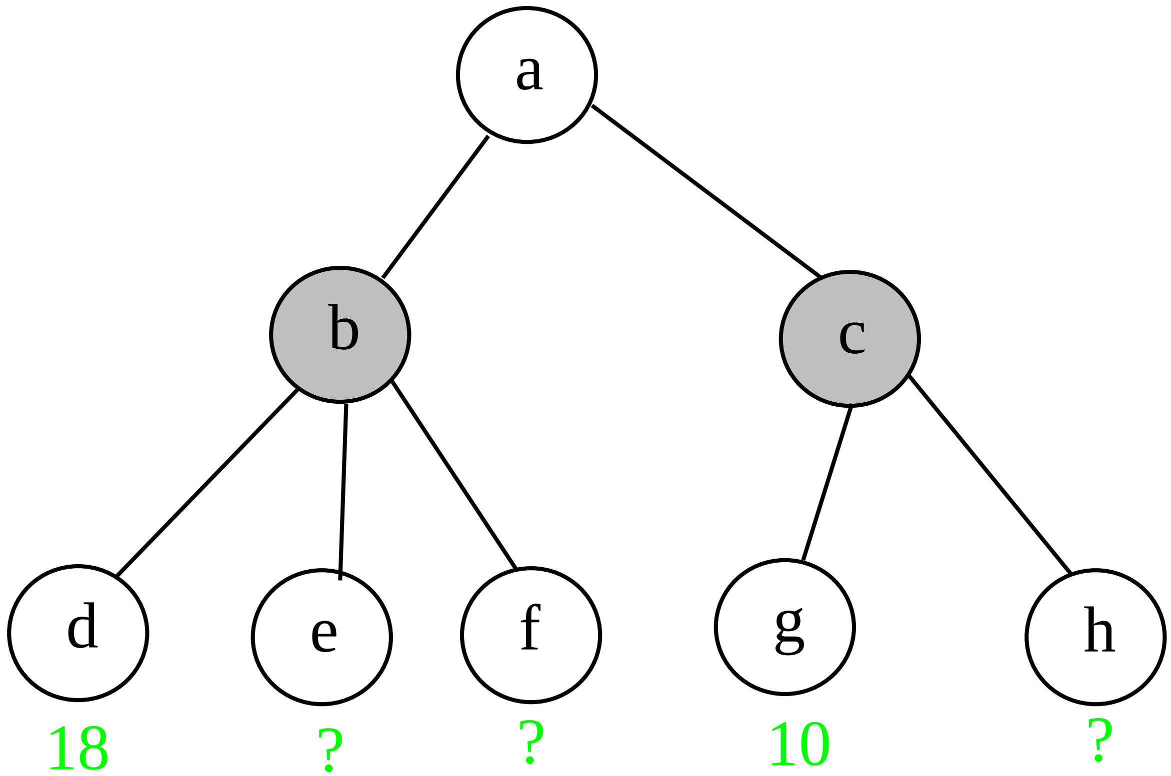
- {$*$ **Continued from the last page** $*$}
  - **solve** $u$;
  - **repeat** {$*$ **bottom up updating the values** $*$}
    - ▷ ***update*** $proof(u)$ ***and*** $disproof(u)$
    - ▷ $u \leftarrow u'$***s parent***

    **until** $u$ **is the root**
  - **go to** $loop$;

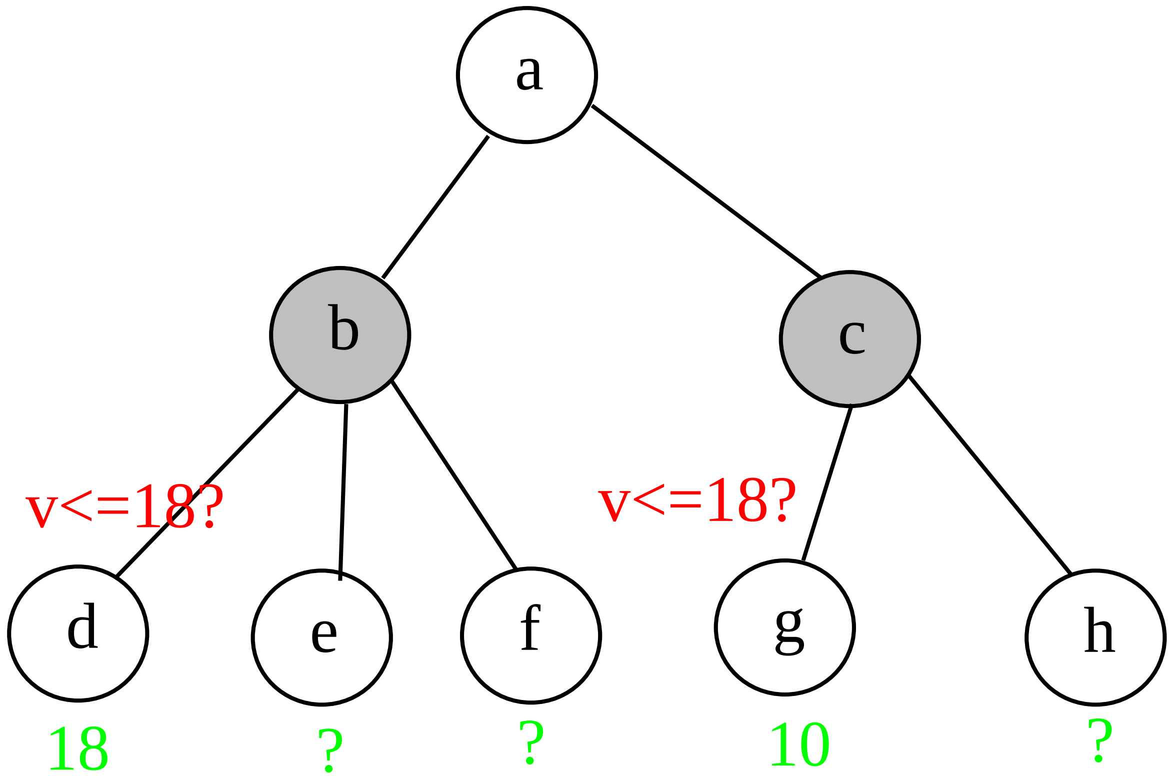
# Multi-Valued game Tree

- **The values of the leaves may not be binary.**
  - **Assume the values are non-negative integers.**
  - **Note: it can be in any finite countable domain.**
- **Revision of the proof and disproof numbers.**
  - $proof_v(u)$: **the minimum number of leaves needed to visited in order for the value of $u$ to $\geq v$.**
    - ▷ $proof(u) \equiv proof_1(u)$.
  - $disproof_v(u)$: **the minimum number of leaves needed to visited in order for the value of $u$ to $< v$.**
    - ▷ $disproof(u) \equiv disproof_1(u)$.

# Illustration



# Illustration



# Multi-Valued proof number

- $u$ **is a leaf:**
  - **If** $value(u)$ **is unknown, then** $proof_v(u)$ **is cost of evaluating** $u$**.**
  - **If** $value(u) \geq v$**, then** $proof_v(u) = 0$**.**
  - **If** $value(u) < v$**, then** $proof_v(u) = \infty$**.**
- $u$ **is an internal node with all of the children** $u_1, \ldots, u_b$**:**
  - **if** $u$ **is a MAX node,**

$$proof_v(u) = \min_{i=1}^{i=b} proof_v(u_i);$$

  - **if** $u$ **is a MIN node,**

$$proof_v(u) = \sum_{i=1}^{i=b} proof_v(u_i).$$

# Multi-Valued disproof number

- $u$ **is a leaf:**
  - **If** $value(u)$ **is unknown, then** $disproof_v(u)$ **is cost of evaluating** $u$**.**
  - **If** $value(u) \geq v$**, then** $disproof_v(u) = \infty$**.**
  - **If** $value(u) < v$**, then** $disproof_v(u) = 0$**.**
- $u$ **is an internal node with all of the children** $u_1, \ldots, u_b$**:**
  - **if** $u$ **is a MAX node,**

$$disproof_v(u) = \sum_{i=1}^{i=b} disproof_v(u_i);$$

  - **if** $u$ **is a MIN node,**

$$disproof_v(u) = \min_{i=1}^{i=b} disproof_v(u_i).$$

# Revised PN-search($v$): algorithm (1/2)

- **{$*$ Compute and update proof$_v$ and disproof$_v$ numbers of the root in a bottom up fashion until it is proved or disproved. $*$}**
- *loop:*
  - **If** $proof_v(root) = 0$ **or** $disproof_v(root) = 0$**, then we are done, otherwise**
    - ▷ $proof_v(root) \leq disproof_v(root)$: ***we try to prove it.***
    - ▷ $proof_v(root) > disproof_v(root)$: ***we try to disprove it.***
  - $u \leftarrow root$; **{$*$ find a leaf to prove or disprove $*$}**
  - **if we try to prove, then**
    - ▷ ***while $u$ is not a leaf do***
    - ▷ ***if $u$ is a MAX node, then***
      ***$u \leftarrow$ leftmost child of $u$ with the smallest non-zero proof$_v$ number;***
    - ▷ ***else if $u$ is a MIN node, then***
      ***$u \leftarrow$ leftmost child of $u$ with a non-zero proof$_v$ number;***
  - **else if we try to disprove, then**
    - ▷ ***while $u$ is not a leaf do***
    - ▷ ***if $u$ is a MAX node, then***
      ***$u \leftarrow$ leftmost child of $u$ with a non-zero disproof$_v$ number;***
    - ▷ ***else if $u$ is a MIN node, then***
      ***$u \leftarrow$ leftmost child of $u$ with the smallest non-zero disproof$_v$ number;***

# PN-search: algorithm (2/2)

- $\{*$ **Continued from the last page** $*\}$
  - **solve** $u$;
  - **repeat** $\{*$ **bottom up updating the values** $*\}$
    - ▷ ***update*** $proof_v(u)$ ***and*** $disproof_v(u)$
    - ▷ $u \leftarrow u'$***s parent***

    **until** $u$ **is the root**
  - **go to** $loop$;

# Multi-valued PN-search: algorithm

- **When the values of the leaves are not binary, use an open value binary search to find an upper bound of the value.**
  - **Set the initial value of $v$ to be 1.**
  - $loop$**: PN-search(**$v$**)**
    - ▷ ***Prove the value of the search tree is*** $\geq v$ ***or disprove it by showing it is*** $< v$.
  - **If it is proved, then double the value of** $v$ **and go to** $loop$ **again.**
  - **If it is disproved, then the true value of the tree is between** $\lfloor v/2 \rfloor$ **and** $v - 1$.
  - **{∗ Use a binary search to find the exact returned value of the tree. ∗}**
  - $low \leftarrow \lfloor v/2 \rfloor$; $high \leftarrow v - 1$;
  - **while** $low \leq high$ **do**
    - ▷ ***if*** $low = high$, ***then return*** $low$ ***as the tree value***
    - ▷ $mid \leftarrow \lfloor (low + high)/2 \rfloor$
    - ▷ ***PN-search***$(mid)$
    - ▷ ***if it is disproved, then*** $high \leftarrow mid - 1$
    - ▷ ***else if it is proved, then*** $low \leftarrow mid$

# Comments

- **Can be used to construct opening books.**
- **Appear to be good for searching certain types of game trees.**
  - **Find the easiest way to prove or disprove a conjecture.**
  - **A dynamic strategy depends on work has been done so far.**
- **Performance has nothing to do with move ordering.**
  - **Performances of most previous algorithms depend heavily on whether good move orderings can be found.**
- **Searching the "easiest" branch may not give you the best performance.**
  - **Performance depends on the value of each internal node.**
- **Commonly used in verifying conjectures, e.g., first-player win.**
  - **Partition the opening moves in a tree-like fashion.**
  - **Try to the "easiest" way to prove or disprove the given conjecture.**
- **Take into consideration the fact that some nodes may need more time to process than the other nodes.**

# More research topics

- **Does a variation of a game make it different?**
  - **Whether Stalemate is draw or win in chess.**
  - **Japanese and Chinese rules in Go.**
  - **Chinese and Asia rules in Chinese chess.**
  - **...**
- **Why a position is easy or difficult to human players?**
  - **Can be used in tutoring or better understanding of the game.**

# Unique features in games

- **Games are used to model real-life problems.**
- **Do unique properties shown in games help modeling real applications?**
  - **Chinese chess**
    - *Very complicated rules for loops: can be draw, win or loss.*
    - *The usage of cannons for attacking pieces that are blocked.*
  - **Go: the rule of Ko to avoid short cycles, and the right to pass.**
  - **Chinese dark chess: a chance node that makes a deterministic ply first, and then followed by a random toss.**
  - **EWN: a chance node that makes a random toss first, and then followed with a deterministic ply later.**
  - **Shogi: the ability to capture an opponent's piece and turn it into your own.**
  - **Chess: stalemate is draw.**
  - **Promotion: a piece may turn into a more/less powerful one once it satisfies some pre-conditions.**
    - *Chess*
    - *Shogi*
    - *Chinese chess: the mobility of a pawn is increased once it advances twice, but is decreased once it reaches the end of a column.*

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