Scout and NegaScout

Tsan-sheng Hsu

徐讚昇

tshsu@iis.sinica.edu.tw

`http://www.iis.sinica.edu.tw/~tshsu`

Abstract

- It looks like alpha-beta pruning is the best we can do for an exact generic searching procedure.
  - What else can be done generically?
  - Alpha-beta pruning follows basically the "intelligent" searching behaviors used by human when domain knowledge is not involved.
  - Can we find some other "intelligent" behaviors used by human during searching?
- Intuition: MAX node.
  - Suppose we know currently we have a way to gain at least 300 points at the first branch.
  - If there is an efficient way to know the second branch is at most gaining 300 points, then there is no need to search the second branch in detail.
    - ▷ *Alpha-beta cut algorithm is one way to make sure of this by returning an exact value.*
    - ▷ *Is there a way to search a tree by only returning a bound?*
    - ▷ *Is searching with a bound faster than searching exactly?*
- Similar intuition holds for a MIN node.

SCOUT procedure

- **It may be possible to verify whether the value of a branch is greater than a value v or not in a way that is faster than knowing its exact value [Judea Pearl 1980].**
- **High level idea:**
  - **While searching a branch T_i of a MAX node, if we have already obtained a lower bound v_ℓ.**
    - ▷ ***First TEST whether it is possible for T_i to return something greater than v_ℓ.***
    - ▷ ***If FALSE, then there is no need to search T_i.***
      $\Rightarrow$ This is called fails the test.
    - ▷ ***If TRUE, then search T_i.***
      $\Rightarrow$ This is called passes the test.
  - **While searching a branch T_j of a MIN node, if we have already obtained an upper bound v_u**
    - ▷ ***First TEST whether it is possible for T_j to return something smaller than v_u.***
    - ▷ ***If FALSE, then there is no need to search T_j.***
      $\Rightarrow$ This is called fails the test.
    - ▷ ***If TRUE, then search T_j.***
      $\Rightarrow$ This is called passes the test.

How to TEST $> v$

procedure TEST$_>$(position p, value v)
// test whether the value of the branch at p is $> v$

- determine the successor positions $p_1, \ldots, p_b$ of p
- if $b = 0$, then // terminal
  - ▷ *if $f(p) > v$ then // f(): evaluation function*
  - ▷ *return TRUE*
  - ▷ *else return FALSE*
- if p is a MAX node, then
  - for $i := 1$ to b do
    - ▷ *if TEST$_>$(p_i, v) is TRUE, then*
      return TRUE // succeed if a branch is $> v$
  - return FALSE // fail only if all branches $\leq v$
- if p is a MIN node, then
  - for $i := 1$ to b do
    - ▷ *if TEST$_>$(p_i, v) is FALSE, then*
      return FALSE // fail if a branch is $\leq v$
  - return TRUE // succeed only if all branches are $> v$

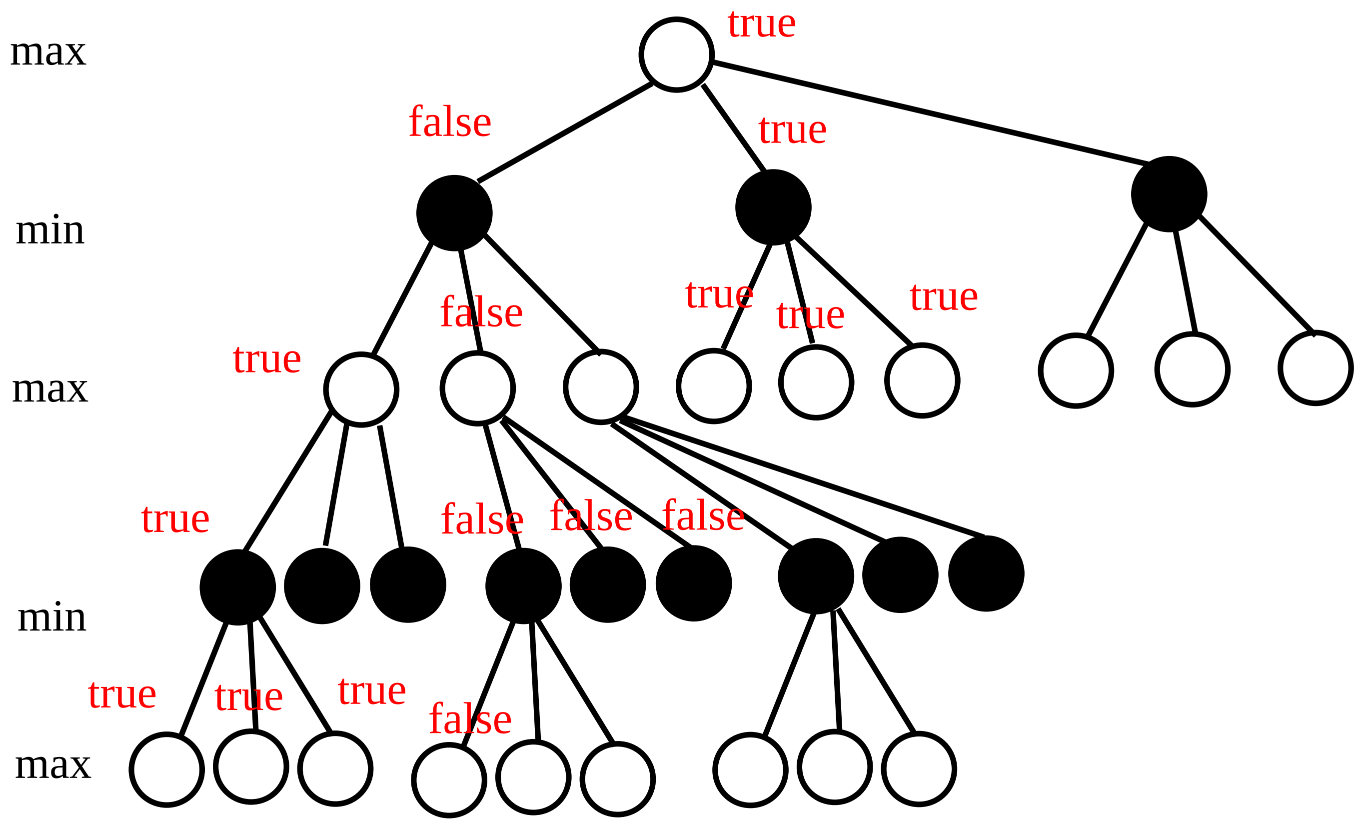
How to TEST $< v$

procedure TEST$_<$(position p, value v)

// test whether the value of the branch at p is $< v$

- **determine the successor positions $p_1, \ldots, p_b$ of p**
- **if $b = 0$, then // terminal**
  - ▷ ***if $f(p) < v$ then // f(): evaluation function***
  - ▷ ***return TRUE***
  - ▷ ***else return FALSE***
- **if p is a MAX node, then**
  - **for $i := 1$ to b do**
    - ▷ ***if TEST$_<(p_i, v)$ is FALSE, then***
      return FALSE // fail if a branch is $\geq v$
  - **return TRUE // succeed only if all branches $< v$**
- **if p is a MIN node, then**
  - **for $i := 1$ to b do**
    - ▷ ***if TEST$_<(p_i, v)$ is TRUE, then***
      return TRUE // succeed if a branch is $< v$
  - **return FALSE // fail only if all branches are $\geq v$**

Illustration of TEST$_{>}$



Short circuit operations for $\text{TEST}_{>}$

- **For a MAX node:**
  - **if a branch is TRUE, then there is no need to do further testing;**
  - **if a branch is FALSE, then we need to do more testing on other branches.**
  - **It is better to test branches with better probabilities of being TRUE first.**
- **For a MIN node:**
  - **if a branch is FALSE, then there is no need to do further testing;**
  - **if a branch is TRUE, then we need to do more testing on other branches.**
  - **It is better to test branches with better probabilities of being FALSE first.**

How to TEST — Discussions

- **Sometimes it may be needed to test for "$\geq v$", or "$\leq v$".**
  - $\boxed{\textbf{TEST}_{>}(p,v) \textbf{ is TRUE}} \equiv \boxed{\textbf{TEST}_{\leq}(p,v) \textbf{ is FALSE}}$
  - $\boxed{\textbf{TEST}_{>}(p,v) \textbf{ is FALSE}} \equiv \boxed{\textbf{TEST}_{\leq}(p,v) \textbf{ is TRUE}}$
  - $\boxed{\textbf{TEST}_{<}(p,v) \textbf{ is TRUE}} \equiv \boxed{\textbf{TEST}_{\geq}(p,v) \textbf{ is FALSE}}$
  - $\boxed{\textbf{TEST}_{<}(p,v) \textbf{ is FALSE}} \equiv \boxed{\textbf{TEST}_{\geq}(p,v) \textbf{ is TRUE}}$
- **Practical consideration:**
  - **Set a depth limit and evaluate the position's value when the limit is reached.**

Main SCOUT procedure

Algorithm SCOUT(position p)

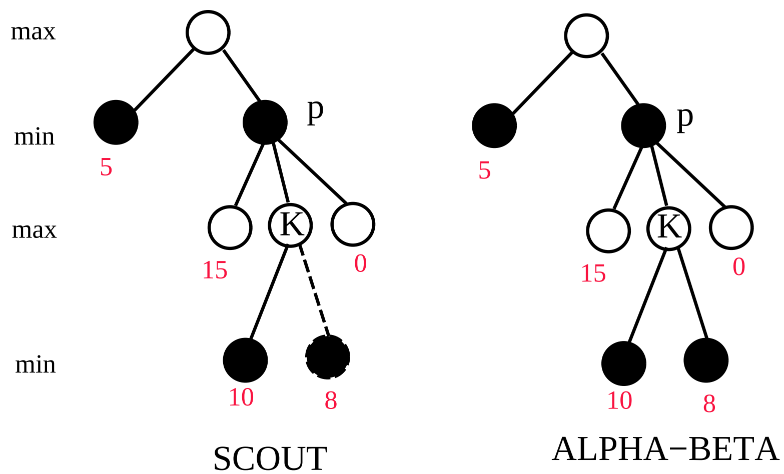
- determine the successor positions $p_1, \ldots, p_b$
- if $b = 0$, then return $f(p)$
- else $v = SCOUT(p_1)$ // SCOUT the first branch
- if p is a MAX node
  - for $i := 2$ to b do
    - *if TEST$_>(p_i, v)$ is TRUE, // TEST first for the rest of the branches*
      then $v = SCOUT(p_i)$ // find the value of this branch if it can be $> v$
- if p is a MIN node
  - for $i := 2$ to b do
    - *if TEST$_<(p_i, v)$ is TRUE, // TEST first for the rest of the branches*
      then $v = SCOUT(p_i)$ // find the value of this branch if it can be $< v$
- return v

Discussions for SCOUT (1/3)

- **Initially, we use recursive call to find the value v of the first branch.**
- **From now on, v is the current best value at any moment.**
- **MAX node:**
  - **For any $i > 1$, if TEST$_>$(p_i, v) is TRUE,**
    - ▷ ***then the value returned by $SCOUT(p_i)$ must be greater than v;***
    - ▷ ***and make this the new v.***
  - **We say that p_i passes the test if TEST$_>$(p_i, v) is TRUE.**
- **MIN node:**
  - **For any $i > 1$, if TEST$_<$(p_i, v) is TRUE,**
    - ▷ ***then the value returned by $SCOUT(p_i)$ must be smaller than v;***
    - ▷ ***and make this the new v.***
  - **We say that p_i passes the test if TEST$_<$(p_i, v) is TRUE.**

Discussions for SCOUT (2/3)

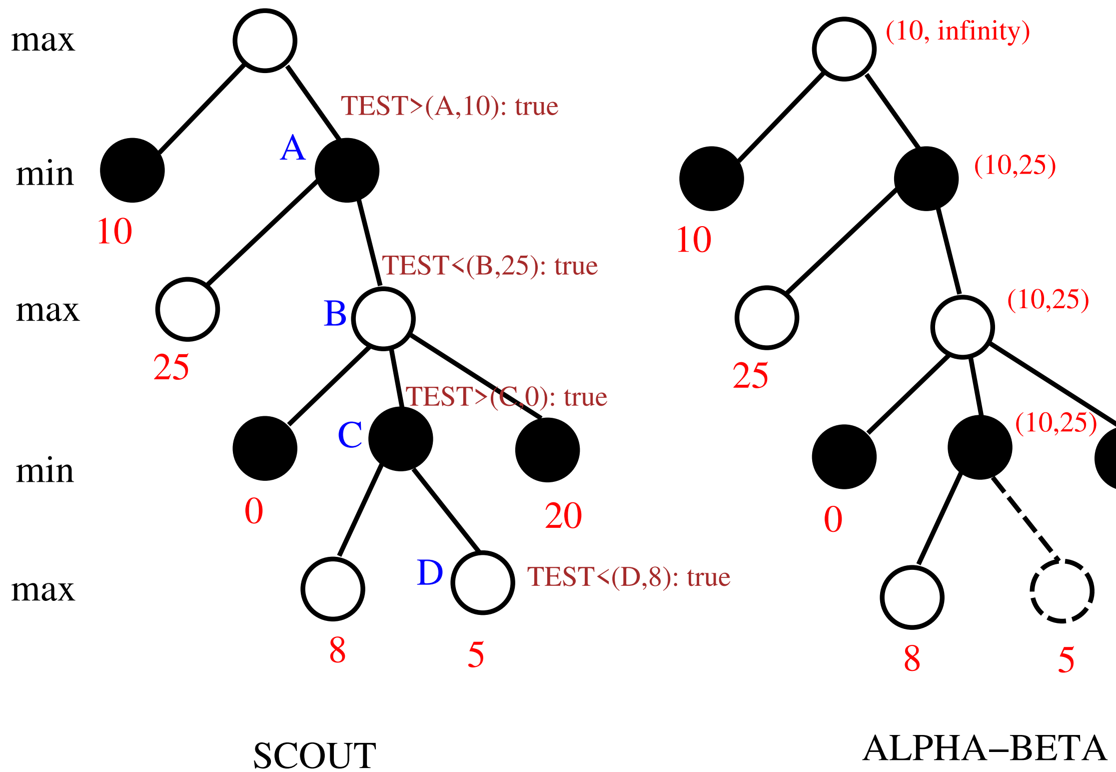
- **TEST which is called by SCOUT may visit less nodes than that of alpha-beta.**



  - **Assume TEST$_>$(p,5) is called by the root after the first branch of the root is evaluated.**
    - ▷ ***It calls TEST$_>$(K,5) which skips K's second branch.***
    - ▷ ***TEST$_>$(p,5) is FALSE, i.e., fails the test, after returning from the 3rd branch.***
    - ▷ ***No need to do SCOUT for the branch rooted p.***
  - **Alpha-beta needs to visit K's second branch.**

Discussions for SCOUT (3/3)

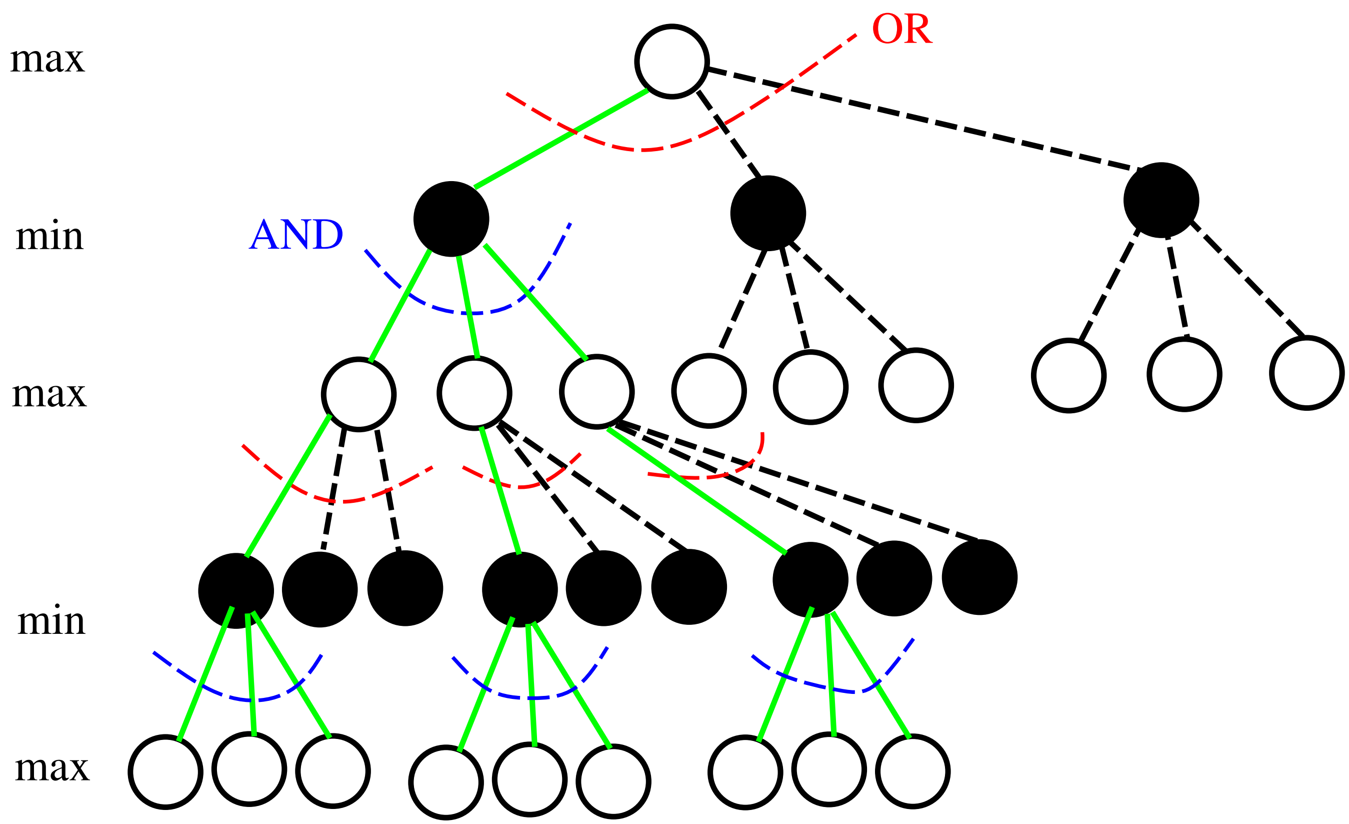
- **SCOUT may pay many visits to a node that is cut off by alpha-beta.**



Number of nodes visited (1/4)

- **For TEST to return TRUE for a subtree T, it needs to evaluate at least**
  - *one child for a MAX node in T, and*
  - *and all of the children for a MIN node in T.*
  - *If T has a fixed branching factor b and uniform depth b, the number of nodes evaluated is $\Omega(b^{\ell/2})$ where ℓ is the depth of the tree.*
- **For TEST to return FALSE for a subtree T, it needs to evaluate at least**
  - *one child for a MIN node in T, and*
  - *and all of the children for a MAX node in T.*
  - *If T has a fixed branching factor b and uniform depth b, the number of nodes evaluated is $\Omega(b^{\ell/2})$.*

Number of nodes visited (2/4)



Number of nodes visited (3/4)

- **Assumptions:**
  - **Assume a full complete b-ary tree with depth ℓ.**
  - **The depth of the root, which is a MAX node, is 0.**
  - **Assume ℓ is even in the analysis.**
- **The total number of nodes in the tree is $\frac{b^{\ell+1}-1}{b-1}$.**
- **H_1: the minimum number of nodes visited by TEST when it returns TRUE.**

$$\begin{aligned} H_1 = &\quad 1 + 1 + b + b + b^2 + b^2 + b^3 + b^3 + \cdots + b^{\ell/2-1} + b^{\ell/2-1} + b^{\ell/2} \\ = &\quad 2 \cdot (b^0 + b^1 + \cdots + b^{\ell/2}) - b^{\ell/2} \\ = &\quad 2 \cdot \frac{b^{\ell/2+1}-1}{b-1} - b^{\ell/2} \end{aligned}$$

Number of nodes visited (4/4)

- **Assumptions:**
  - **Assume a full complete b-ary tree with depth ℓ.**
  - **The depth of the root, which is a MAX node, is 0.**
  - **Assume ℓ is even in the analysis.**
- H_2**: the minimum number of nodes visited by alpha-beta.**

$$\begin{aligned}
H_2 &= \sum_{i=0}^{\ell}(b^{\lceil i/2\rceil} + b^{\lfloor i/2\rfloor} - 1) \\
&= \sum_{i=0}^{\ell} b^{\lceil i/2\rceil} + \sum_{i=0}^{\ell} b^{\lfloor i/2\rfloor} - (\ell+1) \\
&= \sum_{i=0}^{\ell} b^{\lceil i/2\rceil} + H_1 - (\ell+1) \\
&= (1 + b + b + \cdots + b^{\ell/2-1} + b^{\ell/2} + b^{\ell/2}) + H_1 - (\ell+1) \\
&= (H_1 - 1 + b^{\ell/2}) + H_1 - (\ell+1) \\
&= 2 \cdot H_1 + b^{\ell/2} - (\ell+2) \\
&\geq 2 \cdot H_1 \text{ if } b > 3
\end{aligned}$$

Comparisons

- When the first branch of a node has the best value, then TEST scans the tree fast.
  - The best value of the first $i-1$ branches is used to test whether the ith branch needs to be searched exactly.
  - If the value of the first $i-1$ branches of the root is better than the value of ith branch, then we do not have to evaluate exactly for the ith branch.
- Compared to alpha-beta pruning whose cut off comes from bounds of search windows.
  - It is possible to have some cut-off for alpha-beta pruning as long as some relative move orderings are "good."
    - ▷ *The moving orders of your children and the children of your ancestor who is odd level up "together" decide a cut-off.*
  - The bounds are updated during searching.
    - ▷ *Sometimes, a deep alpha-beta cut-off occurs because a bound found from your ancestor a distance away.*

Performance of SCOUT (1/3)

- **A node may be visited more than once.**
  - **First visit is to TEST.**
  - **Second visit is to SCOUT.**
    - ▷ *During SCOUT, it may be TESTed with a different value.*
  - **Q: Can information obtained in the first search be used in the second search?**
- **SCOUT is a recursive procedure.**
  - **For every node v in a branch that is not the first visited child of its parent with a depth[1] of ℓ,**
    - ▷ *every ancestor of v may initiate a TEST to visit v.*
    - ▷ *It can be visited ℓ times by TEST.*

[1] The depth of the root is defined to be 0.

Performance of SCOUT (2/3)

- **Show great improvements on $depth > 3$ over brute-force methods for games with small branching factors.**
  - **It traverses most of the nodes without evaluating them preciously.**
  - **Few subtrees remained to be revisited to compute their exact mini-max values.**
- **Show good improvement over alpha-beta on game trees with certain characteristics.**
- **Experimental data on the game of Kalah show [UCLA Tech Rep UCLA-ENG-80-17, A comparison of the Alpha-Beta and SCOUT algorithms using the game of Kalah, Noe 1980]:**
  - **SCOUT favors "skinny" game trees, that are game trees with high depth-to-width ratios.**
    - ▷ ***Q: why?***
  - **On depth $= 5$, it saves over 40% of time.**
  - **May not be good for games with large branching factors.**
  - **Move ordering is very important.**
    - ▷ ***The first branch, if is good, offers a great chance of pruning further branches.***

Performance of SCOUT (3/3)

- **Comparing alpha-beta pruning and SCOUT [Pearl 1984] on uniform game trees:**
  - **Alpha-beta is always better than SCOUT in the experiments using random game trees.**
    - ▷ ***In theory, when both are in their best cases, SCOUT cuts out more, but this rarely happens in practice.***
  - **Let $r_{b,d} = \frac{N_{scout}}{N_{AB}}$ where N_{scout} is the nodes searched using SCOUT and N_{AB} is the nodes searched using alpha-beta on depth-d random-valued game trees with a uniform branching factor of b.**
    - ▷ $1 \leq r_{b,d} \leq 1.275$ ***for any positive integers*** b ***and*** d.
    - ▷ $r_{b_1,d} \geq r_{b_2,d}$ ***if*** $b_1 \leq b_2$: ***ratio is closer when the branching factor is larger.***
    - ▷ $r_{b,d_1} \geq r_{b,d_2}$ ***if*** $d_1 \leq d_2$: ***ratio is closer when the searching depth is larger.***
    - ▷ $r_{2,20} \sim 1.04$.
    - ▷ $r_{b,20} \sim 1$: ***after depth*** > 20, ***the two are almost the same.***

Comments

- **Q1:**
  - **Currently, we use a "feasible" test to decide whether we need to search this branch or not.**
    - *If a new branch has a chance of larger than v, then we explore it in details. Otherwise, we skip it.*
  - **How about using the idea of "infeasible" test?**
    - *If a new branch has no chance of larger than v, then we do not explore it in details. Otherwise, we do.*
  - **How about a hybrid approach?**
    - *When to use one instead of the other?*
- **Q2: What can we do with regard to the first branch?**
  - **Can some previous values of some previous positions be used?**
  - **When iterative deepening is used, can we use previous results?**

Alpha-beta revisited

- **In an alpha-beta search with a window ($alpha$,$beta$):**
  - **Failed-high** means it returns a value that is larger than or equal to its upper bound $beta$.
  - **Failed-low** means it returns a value that is smaller than or equal to its lower bound $alpha$.
- **Null or Zero window search:**
  - Using alpha-beta search with the window (m,$m+1$).
    - ▷ *Can never happen in a normal alpha-beta pruning when starts with ($-\infty$,∞).*
  - The result can be either failed-high or failed-low.
  - Failed-high means the return value is at least $m+1$.
    - ▷ *Equivalent to $TEST_>(p,m)$ is TRUE.*
  - Failed-low means the return value is at most m.
    - ▷ *Equivalent to $TEST_>(p,m)$ is FALSE.*
- **The above argument works for the shallow fail hard ($F1$), general fail hard ($F2$) and general fail soft ($F3$) versions of the alpha-beta algorithm.**

Behaviors of Null window search

- When $F2(p, m, m+1, \infty)$ returns $m+1$:
  - for the MAX node p, returns immediately after the first child p_i, namely the smallest index i, returning a value $\geq m+1$.
  - for the MIN node p_i, every child $p_{i,j}$ returns a value $\geq m+1$
  - for each MAX node $p_{i,j}$, returns immediately after the first child $r_{i,j,k}$, namely the smallest index k, returning a value $\geq m+1$.
  - $\ldots$
  - **Remark:** $F3(p, m, m+1, \infty)$ returns a value $\geq m+1$ in this case.
- Exactly like the OR-AND tree shown in TEST$_>$ when TEST is passed.
- We can observe similar behaviors when $F2(p, m, m+1, \infty)$ returns m as if TEST is failed.
  - **Remark:** $F3(p, m, m+1, \infty)$ returns a value $\leq m$ in this case.

Alpha-Beta + Scout

- **Intuition:**
  - Try to incooperate SCOUT and alpha-beta together.
  - The searching window of alpha-beta if properly set can be used as TEST in SCOUT.
  - Using a searching window is better than using a single bound as in SCOUT.
  - Can also apply alpha-beta cut if it applies.
- **Modifications to the SCOUT algorithm:**
  - Traverse the tree with two bounds as the alpha-beta procedure does.
    - ▷ *A searching window.*
    - ▷ *Use the current best bound to guide the value used in TEST.*
  - Use a fail soft version to get a better result when the returned value is out of the window.

The NegaScout Algorithm – Mini-Max (1/2)

- **Algorithm $F4'$(position p, value $alpha$, value $beta$, integer $depth$)**
  - determine the successor positions $p_1, \ldots, p_b$
  - if $b = 0$ // a terminal node
    or $depth = 0$ // $depth$ is the remaining depth to search
    or time is running up // from timing control
    or some other constraints are met // apply heuristic here
  - then return $f(p)$ else
    begin
    - ▷ $m := -\infty$ // m *is the current best lower bound; fail soft*
      $m := \max\{m, G4'(p_1, alpha, beta, depth - 1)\}$ // *the first branch*
      if $m \geq beta$ *then return*(m) // *beta cut off*
    - ▷ *for* $i := 2$ *to* b *do*
    - ▷ $t := G4'(p_i, m, m + 1, depth - 1)$ // *null window search*
    - ▷ *if* $t > m$ *then* // *failed-high*
      if $(depth < 3$ *or* $t \geq beta)$
      then $m := t$
      else $m := G4'(p_i, t, beta, depth - 1)$ // *re-search*
    - ▷ *if* m *is max possible or* $m \geq beta$ *then return*(m) // *beta cut off*

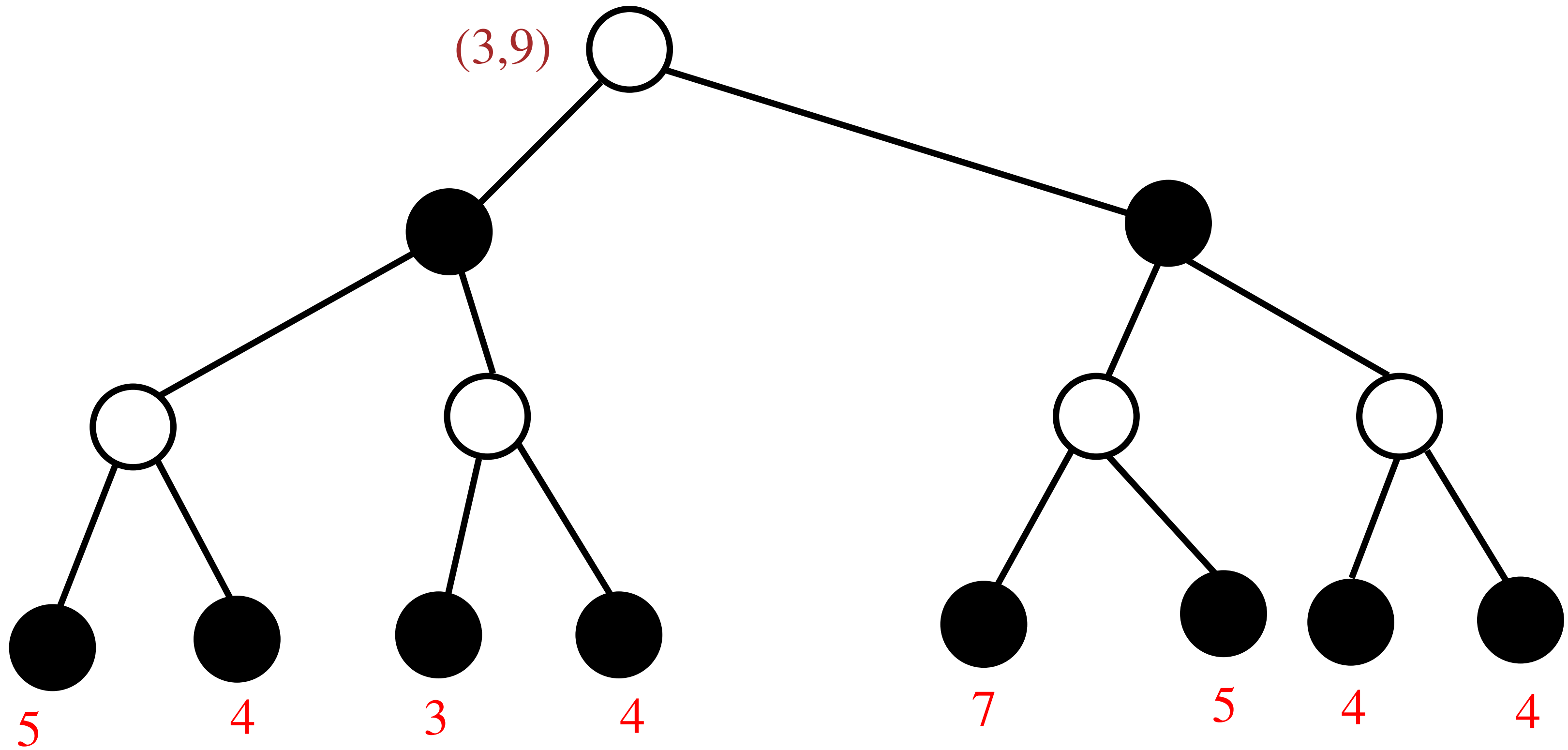
    end
  - return m

The NegaScout Algorithm – Mini-Max (2/2)

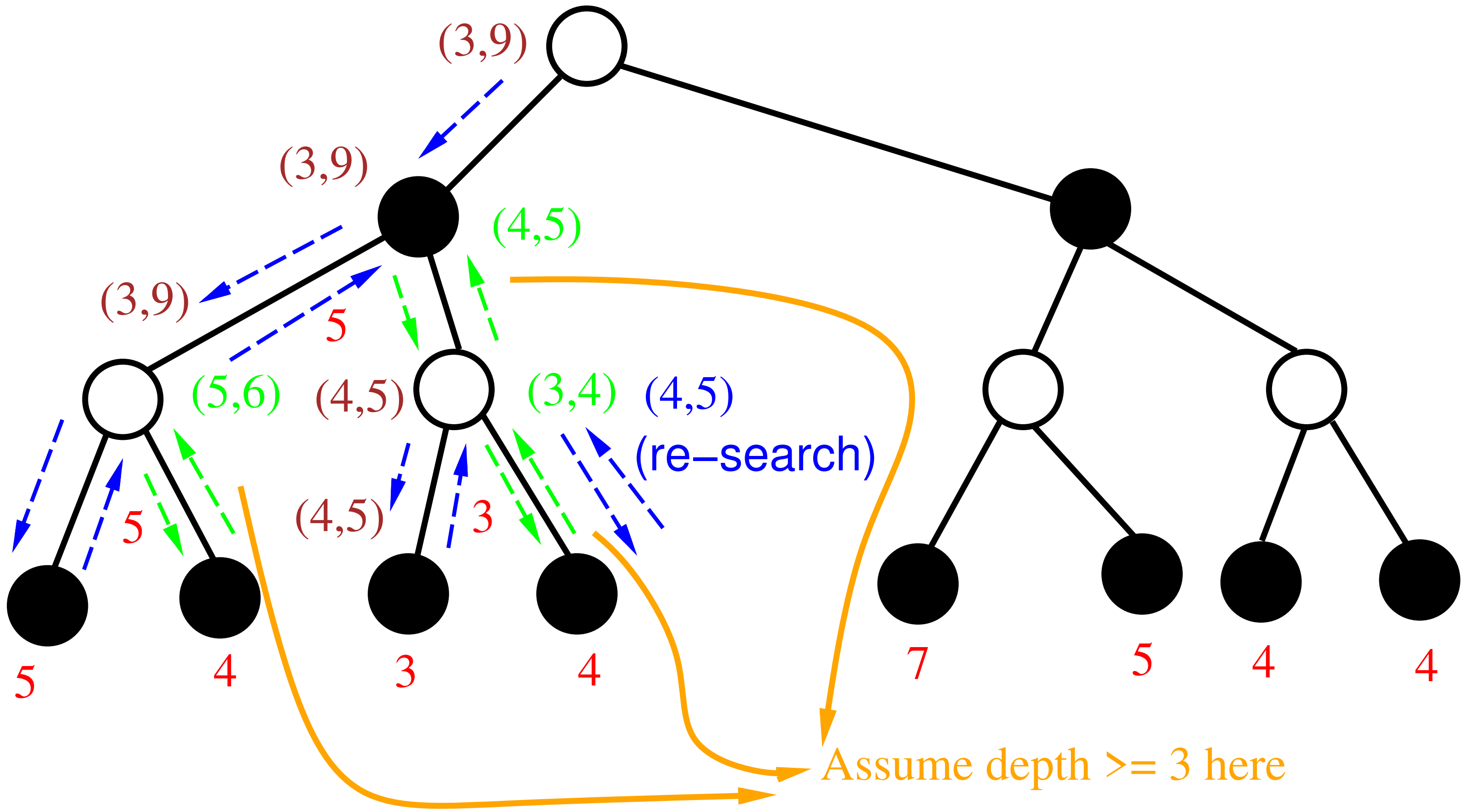
- **Algorithm** $G4'$(**position** p, **value** $alpha$, **value** $beta$, **integer** $depth$)
  - determine the successor positions $p_1, \ldots, p_b$
  - if $b = 0$ // a terminal node
    or $depth = 0$ // $depth$ is the remaining depth to search
    or time is running up // from timing control
    or some other constraints are met // apply heuristic here
  - then return $f(p)$ else
    begin
    - ▷ $m = \infty$ // m *is the current best upper bound; fail soft*
      $m := \min\{m, F4'(p_1, alpha, beta, depth-1)\}$ // *the first branch*
      if $m \leq alpha$ *then return*(m) // *alpha cut off*
    - ▷ *for* $i := 2$ *to* b *do*
    - ▷ $t := F4'(p_i, m-1, m, depth-1)$ // *null window search*
    - ▷ *if* $t < m$ *then* // *failed-low*
      if ($depth < 3$ *or* $t \leq alpha$)
      then $m := t$
      else $m := F4'(p_i, alpha, t, depth-1)$ // *re-search*
    - ▷ *if* m *is min possible or* $m \leq alpha$ *then return*(m)// *alpha cut off*

    end
  - return m

NegaScout – Mini-Max version (1/2)



NegaScout – Mini-Max version (2/2)



The NegaScout Algorithm

- **Use Nega-MAX format.**
- **Algorithm $F4$(position p, value $alpha$, value $beta$, integer $depth$)**
  - determine the successor positions $p_1, \ldots, p_b$
  - if $b = 0$ // a terminal node
    or $depth = 0$ //$depth$ is the remaining depth to search
    or time is running up // from timing control
    or some other constraints are met // apply heuristic here
  - then return $h(p)$ else
    - ▷ $m := -\infty$ // *the current lower bound; fail soft*
    - ▷ $n := beta$ // *the current upper bound*
    - ▷ ***for*** $i := 1$ ***to*** b ***do***
    - ▷ $t := -F4(p_i, -n, -max\{alpha, m\}, depth - 1)$
    - ▷ ***if*** $t > m$ ***then***
      if $(n = beta$ ***or*** $depth < 3$ ***or*** $t \geq beta)$
      then $m := t$
      else $m := -F4(p_i, -beta, -t, depth - 1)$ // ***re-search***
    - ▷ ***if*** m ***is max possible or*** $m \geq beta$ ***then return***(m) // ***cut off***
    - ▷ $n := max\{alpha, m\} + 1$ // ***set up a null window***
  - return m

Search behaviors (1/3)

- **If the depth is enough or it is a terminal position, then stop searching further.**
  - **Return $h(p)$ as the value computed by an evaluation function.**
  - **Note:**

$$h(p) = \begin{cases} f(p) & \textbf{if depth of } p \textbf{ is 0 or even} \\ -f(p) & \textbf{if depth of } p \textbf{ is odd} \end{cases}$$

- **Fail soft version.**
- **Search the first child p_1 using the normal alpha beta window.**
  - **line 9: normal window for the first child**
    - ***the initial value of*** m ***is*** $-\infty$***, hence*** $-max\{alpha, m\} = -alpha$
    - m ***is the current best value***
    - ***that is, equivalent to***

      9: $t := -F4(p_i, -beta, -alpha, depth - 1)$

      searching with the normal window $(alpha, beta)$

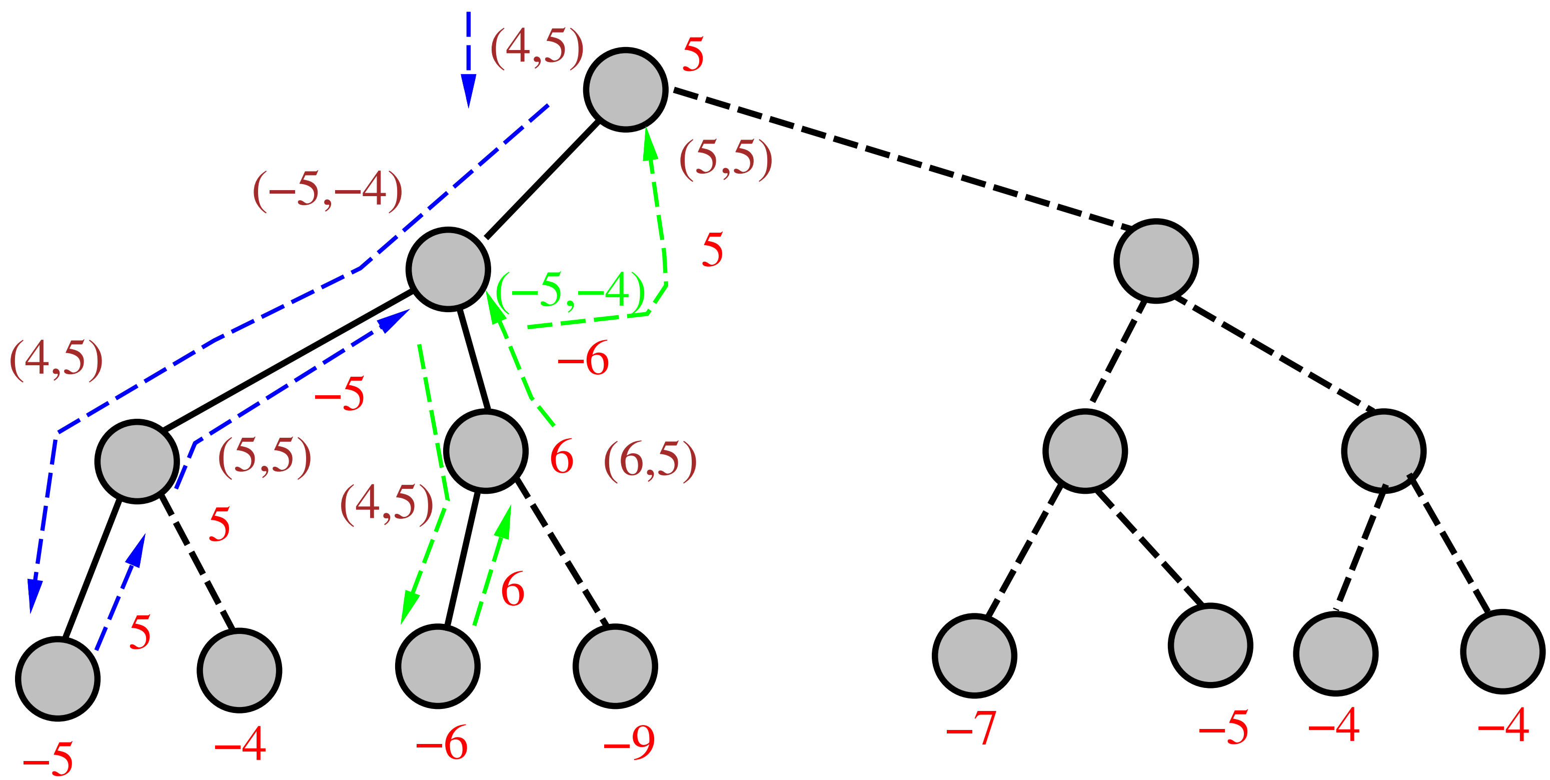
Search behaviors (2/3)

- **For the second child and beyond p_i, $i > 1$, first perform a null window search for testing whether m is the answer.**
  - **line 9: a null-window of $(n-1, n)$ searches for the second child and beyond where $n = max\{alpha, m\} + 1$.**
    - ▷ ***m is best value obtained so far***
    - ▷ ***$alpha$ is the previous lower bound***
    - ▷ ***m's value will be first set at line 12 because $n = beta$***
    - ▷ ***The value of $n = max\{alpha, m\} + 1$ is set at line 15.***
  - **line 11:**
    - ▷ ***If $n = beta$, we are at the first iteration.***
    - ▷ ***If $depth < 3$, we are on a smaller depth subtree, i.e., depth at most 2, NegaScout always returns the best value.***
    - ▷ ***If $t \geq beta$, we have obtained a good enough value from the failed-soft version to guarantee a beta cut.***

Search behaviors (3/3)

- **For the second child and beyond p_i, $i > 1$, first perform a null window search for testing whether m is the answer.**
  - **line 11: on a smaller depth subtree, i.e., depth at most 2, NegaScout always returns the best value.**
    - ▷ *Normally, no need to do alpha-beta or any enhancement on very small subtrees.*
    - ▷ *The overhead is too large on small subtrees.*
  - **line 13: re-search when the null window search fails high.**
    - ▷ *The value of this subtree is at least t.*
    - ▷ *This means the best value in this subtree is more than m, the current best value.*
    - ▷ *This subtree must be re-searched with the the window $(t, beta)$.*
  - **line 14: the normal pruning from alpha-beta.**

Example for NegaScout



Refinements

- When a subtree is re-searched, it is best to use information on the previous search to speed up the current search.
  - Restart from the position that the value t is returned.
- Maybe want to re-search using the normal alpha-beta procedure.
- $F4$ runs much better with a good move ordering and some form of a transposition table which will be introduced later.
  - Order the moves in a priority list.
  - Reduce the number of re-searching's.

Performances

- **Experiments done on a uniform random game tree [Reinefeld 1983].**
  - **Normally superior to alpha-beta when searching game trees with branching factors from 20 to 60.**
  - **Shows about 10 to 20% of improvement.**

Comments

- Incooperating both SCOUT and alpha-beta.
- Used in state-of-the-art game search engines.
- The first search, though maybe unsuccessful, can provide useful information in the second search.
  - Information can be stored and then reused.
- Using TEST in SCOUT to do the first search because it has a chance to visit less nodes than that of ALPHA-BETA.

References and further readings

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