

Two-Player Perfect Information Games: A Brief Survey

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Abstract

- Domain: two-player games.
- Which game characters are predominant when the solution of a game is the main target?
 - It is concluded that **decision complexity** is more important than **state-space complexity**.
 - There is a trade-off between using **knowledge-based methods** and using **brute-force methods**.
 - There is a clear correlation between the first-player's **initiative** and the necessary effort to solve a game.
 - **Fairness** of a game.
- A survey of related studies on two-player games.
 - Mathematical properties.
 - ▷ *Strategy-stealing argument.*
 - Computational results.

Domain of studies

- Domain: 2-person **zero-sum** games with **perfect information**.
 - Result: **win**, **loss** or **draw**.
 - ▷ *Some games, such as EWN, have no draw.*
 - **Zero-sum** means one player's loss is exactly the other player's gain, and vice versa.
 - ▷ *There is no way for both players to win at the same time.*
 - ▷ *By the same token, there is no way for both players to lose at the same time.*
 - ▷ *In these cases, it is usually called a **tie**¹ or **draw**²*

¹ "Tie" sometimes means both sides have chance to win.

² "Draw" sometimes means both sides have no chance to win.

Positions

- An arrangement of pieces on the board with **an indication of who the next player is**.
- Sometimes a position includes some historical information such the location of Ko in Go.
- Classifications
 - **Legal** position: a position that can be reached from the initial position(s).
 - ▷ *Note this is different from forming all possible arrangements using rules.*
 - ▷ *For example: When 32 pieces are left in a chess board, it is impossible to have 3 pawns of one color to be in one column.*
 - Initial position(s)
 - Goal position(s)

Unreasonable, but Legal Positions

■ **Unreasonable**, but legal, positions

- Positions that are unlikely to happen in real tournaments.
- Examples
 - ▷ *I: In chess, all of your pieces are alive while the opponent has only the king left.*
 - ▷ *II: if one can capture the opponent's king in a position and thus wins in chess, but he makes another move so that another piece also checks the king, then it results in a double checked position which is unreasonable.*
- How unreasonable is unreasonable is a question that is similar to how difficult a position is. It could be results of player(s) making mistake(s).

Game-theoretic value

- **Game-theoretic value** of a game: the outcome, i.e., win, loss or draw, when all participants play optimally.
- Classification of games' solutions according to L.V. Allis [Ph.D. thesis 1994] if they are considered solved.
 - **Ultra-weakly solved**: the game-theoretic value of the initial position has been determined.
 - **Weakly solved**: for the initial position a strategy has been determined to achieve the game-theoretic value against any opponent.
 - ▷ *Such a strategy is called **optimal**.*
 - ▷ *The strategy must be efficient and practical in terms of resource usage.*
 - ▷ *If the game-theoretical value is win, then the optimal strategy has a role as the first player.*
 - ▷ *If the game-theoretical value is loss, then the optimal strategy has a role as the second player.*
 - ▷ *If the game-theoretical value is draw, then the optimal strategy must be able to **at least** draw any opponent from both roles as the first and the second player.*
 - **Strongly solved**: an optimal strategy has been determined for all legal positions.

Note on game-theoretic value

- The game-theoretic values of most games are **unknown** or are only known for some **legal positions**, in particular the initial position.
- This is one of the most challenging and exciting research areas in games.

Complexity of a game

- **State-space** complexity of a game: the number of **legal positions** in a game.
 - Often it is difficult to filter out unreasonable positions, instead we use all possible arrangements for convenience.
- **Game-tree** (or **decision**) complexity of a game: the number of nodes in a **solution search tree**.
 - Actually, it is usually a game **graph**, not tree.
 - A solution search tree is a tree where the game-theoretic value of the root position can be decided.
 - Each node in the tree is a legal position. The children of a parent node p are the positions that p can reach in one step.
 - ▷ *Some children of a node may not be in a solution search tree.*
 - ▷ *For example, if facing a position, one can capture the opponent's king and thus wins in Chess, but he makes other move and as a result his king is captured in the next ply.*
 - Some legal states may not be in an optimal solution search tree.
 - ▷ *Most, if not all, of them are **unreasonable** positions.*
 - Mostly concerns about weakly solving the game.

More definitions (1/3)

- **Initiative**: the right to move first.
 - Many games are known to be favor to the first player.
 - ▷ *Go-Moku.*
 - Only very few games are known to be favor to the second player.
 - ▷ *6 by 6 Othello.*
- A **fair** game: the game-theoretic value is draw and both players have roughly an equal probability to make a mistake.
 - People normally enjoy playing fair games over unfair ones.
 - Examples:
 - ▷ *Paper-scissor-stone.*
 - ▷ *Roll a dice and the one getting a larger number wins.*
 - ▷ *Nine Men's Morris (weakly proven in 1995; strongly proven in 2010 [Edelkamp et al '10]).*
 - ▷ *Checkers (weakly proven in 2007).*
 - ▷ *8 by 8 Othello (weakly proven in 2024 [Takizawa '24]).*
 - Many popular games are not fair or are unknown of their fairness.
 - It is difficult to prove a non-trivial game is fair or to design a non-trivial fair one, **which is a challenging research area.**

More definitions (2/3)

- An **asymmetric** game
 - An asymmetric game is one that has different rules for the two players.
 - Examples: Renju, Go with a non-zero Komi value.
- A **convergent** game: the size of the state space decreases as the game progresses.
 - Start with many pieces on the board and pieces are gradually removed during the course of the game.
 - ▷ *Example: Checkers.*
 - It means the number of possible configurations decreases as the game progresses.
- A **divergent** game: the size of the state space increases as the game progresses.
 - May start with an empty board, and pieces are gradually added during the course of the game.
 - ▷ *Example: Connect-5 before the board is almost filled.*
 - It means the number of possible configurations increases as the game progresses.
 - ▷ *For Chinese chess, a rook can visit more places when it is away from its initial location.*

More definitions (3/3)

- A game may be convergent at one stage and then divergent at other stage.
 - Most games are dynamic.
 - For the game of Tic-Tac-Toe, assume a total of x plys are played where x is even.
 - ▷ *Then you have a possible of*

$$\binom{9}{x/2} \binom{9 - x/2}{x/2}$$

different configurations.

- ▷ *This number is not monotonously increasing or decreasing.*

Predictions made in 1990

- Predictions were made in 1990 [Allis et al. 1991] for the year 2000 concerning the expected playing strength of computer programs.

solved	over champion	world champion	grand master	amateur
Connect-four	Checkers (8 * 8)	Chess	Go (9 * 9)	Go (19 * 19)
Qubic	Renju	Draughts (10 * 10)	Chinese chess	
Nine Men's Morris	Othello		Bridge	
Go-Moku	Scrabble			
Awari	Backgammon			

- ▷ *Over champion means definitely over the best human player.*
- ▷ *World champion means equaling to the best human player.*
- ▷ *Grand master means beating most human players.*

A double dichotomy of the game space

$\log \log(\text{state-space complexity}) \uparrow$

category 3 if solvable at all, then by knowledge-based methods	category 4 currently unsolvable by any method
category 1 solvable by any method	category 2 if solvable at all, then by brute-force methods

$\log \log(\text{game-tree complexity}) \rightarrow$

Questions to be researched

- Can perfect knowledge obtained from solved games be translated into rules and strategies which human beings can assimilate?
- Are such rules generic, or do they constitute a multitude of ad hoc recipes?
- Can methods be transferred between games?
 - More specifically, are there generic methods for all category- i games, or is each game in a specific category a law unto itself?

Convergent games (1/2)

- Since most games are dynamic, here we consider games whose ending phases are convergent.
 - Can be solved by the method of **endgame databases** if we can enumerate and store all possible positions at a certain stage.
- Problems solved (mostly using **retrograde analysis**):
 - Nine Men's Morris: (weakly) in the year 1995, a total of 7,673,759,269 states after the initial adding phase, and strongly in 2010.
 - ▷ *The game theoretic value is **draw** which means it is a fair game.*
 - Mancala games
 - ▷ *Awari: in the year 2002 whose game theoretic value is **draw**.*
 - ▷ *Kalah: in the year 2011³ upto Kalah(6,6) whose game theoretic value is **first player win**.*

³<http://kalaha.krus.dk/>

Convergent games (2/2)

- More problems solved (mostly using **retrograde analysis**):
 - Checkers
 - ▷ *By combining endgame databases, middle-game databases and verification of opening-game analysis.*
 - ▷ *Solved the so called 100-year position in 1994.*
 - ▷ *The game is weakly proved to be a **draw** in 2007.*
 - ▷ ***A fair game.***
 - Chess endgames
 - Chinese chess endgames
 - Chinese dark chess endgames
 - EWN endgames
 - ...

Divergent games

- Since most games are dynamic, here we consider games whose INITIAL phases are divergent.
- Connection games
 - Connect-four ($6 * 7$)
 - Qubic ($4 * 4 * 4$)
 - Go-Moku ($15 * 15$)
 - Renju
 - k -in-a-row games
 - Hex ($10 * 10$ or $11 * 11$)
- Polyomino games: place pieces inside a board without overlapping and alternatively until one cannot place more.
 - Pentominoes
 - Domineering
- Othello
- Chess
- Chinese chess
- Shogi
- Go

Connection games (1/2)

■ Connect-four ($6 * 7$)

- Weakly solved by J. Allen in 1989 using a brute-force depth first search with alpha-beta pruning, a transposition table, and killer-move heuristics.
- Also weakly solved by L.V. Allis in 1988 using a knowledge-based approach by combining 9 strategic rules that identify potential **threats** of the opponent.
 - ▷ *Threats are something like forced moves or moves you have little choices.*
 - ▷ *Threats are moves with predictable counter-moves.*
- It is first-player-win.
- Weakly solved on a SUN-4 workstation using 300+ hours.

■ Qubic ($4 * 4 * 4$)

- A three-dimensional version of Tic-Tac-Toe.
- Connect-four played on a $4 * 4 * 4$ game board.
- Weakly solved in 1980 by O. Patashnik by combining the usual depth-first search with expert knowledge for ordering the moves.
 - ▷ *It is first-player-win for the 2-player version.*

Connection games (2/2)

■ Go-Moku ($15 * 15$)

- Weakly solved by L.V. Allis in 1995 using a combination of threat-space search and database construction.
- First-player-win.

■ Renju

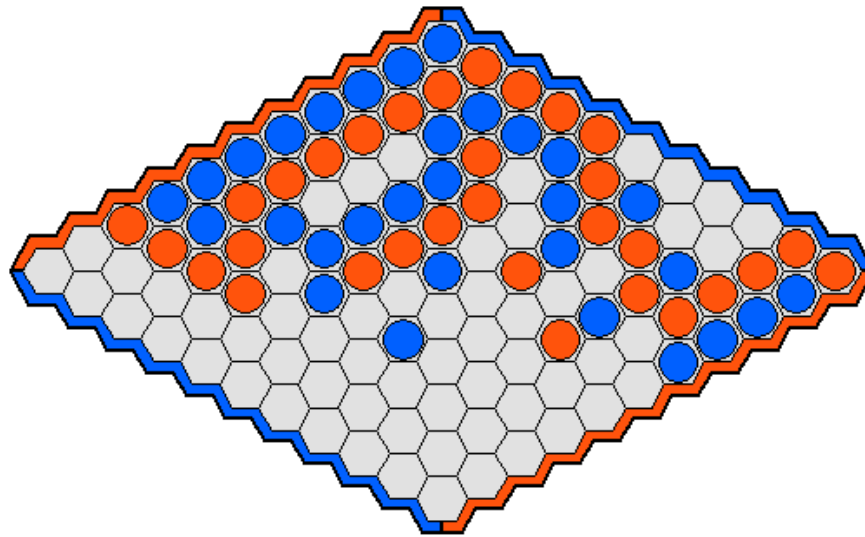
- Does not allow the first player to play certain moves.
- An **asymmetric** game.
- Weakly solved by Wágner and Virág in 2000 by combining search and knowledge.
 - ▷ *Took advantage of an iterative-deepening search based on threat sequences up to 17 plies.*
 - ▷ *It is still first-player-win.*

■ k -in-a-row games

- $m n k$ -Game: a game playing on a board of m rows and n columns with the goal of obtaining a straight line of length k .
- Variations: first ply picks only one stone, the rest of the plys pick two stones at one time.
 - ▷ *Connect6.*
 - ▷ *Try to balance the advantage of the initiative!*

Hex (10 * 10 or 11 * 11)

- Invented in 1942 by Peit Hein and John Hash (source: wiki)
- Rules
 - Two players place one piece of each one's color alternatively.
 - A player connects the NW and SE sides, while the other one connects the NE and SW sides.
 - One who can do the assigned connection wins.



Red won

Courtesy of ICGA web site

HEX: properties

■ Revised rules:

- ▷ *When the first player wins, allow the second player to play one more time. If the second player also wins, then the game is tie.*
- ▷ *When the board is full and no one wins, then it is a draw.*

■ Properties:

- **Property 0: It is a finite game.**
 - ▷ *Trivially true.*
- **Property 1: It is impossible for both players to win at the same time.**
 - ▷ *No chance of tie.*
- **Property 2: There is at least one winner.**
 - ▷ *No chance of draw.*
- **Property 3: It is a first-player-win game.**

Property 1: there cannot be two winners

- A topological argument.
 - A vertical chain can only be cut by a horizontal chain and vice versa because each cell is connected with 6 adjacent cells.
 - ▷ *Note if a cell has 4 neighbors as in the case of Go, then it is possible to cut off a vertical chain by cells that are not horizontally connected and vice versa.*
- Other arguments such as one using graph theory exist.

Property 2: there is at least one winner

- **Assume there is no winner when the board is full.**
- **W.l.o.g. let R be the set of red cells that can be reached by chains originated from the rightmost column.**
 - R must contain a cell of the rightmost column; otherwise we have a contradiction which means the blue wins.
- **Let $N(R)$ be the blue cells that can be reached by R originated from the rightmost column.**
 - $N(R)$ must contain a cell in the top row.
 - ▷ *Otherwise, R contains all cells in the first row, which is a contradiction.*
 - $N(R)$ must contain a cell in the bottom row.
 - ▷ *Otherwise, R contains all cells in the bottom row, which is a contradiction.*
 - $N(R)$ must be connected.
 - ▷ *Otherwise, R can advance further.*
 - Hence $N(R)$ is a blue winning chain.
- **Corollary: only a chain of a direction can stop a chain of the other direction.**

Illustration of the ideas (1/3)

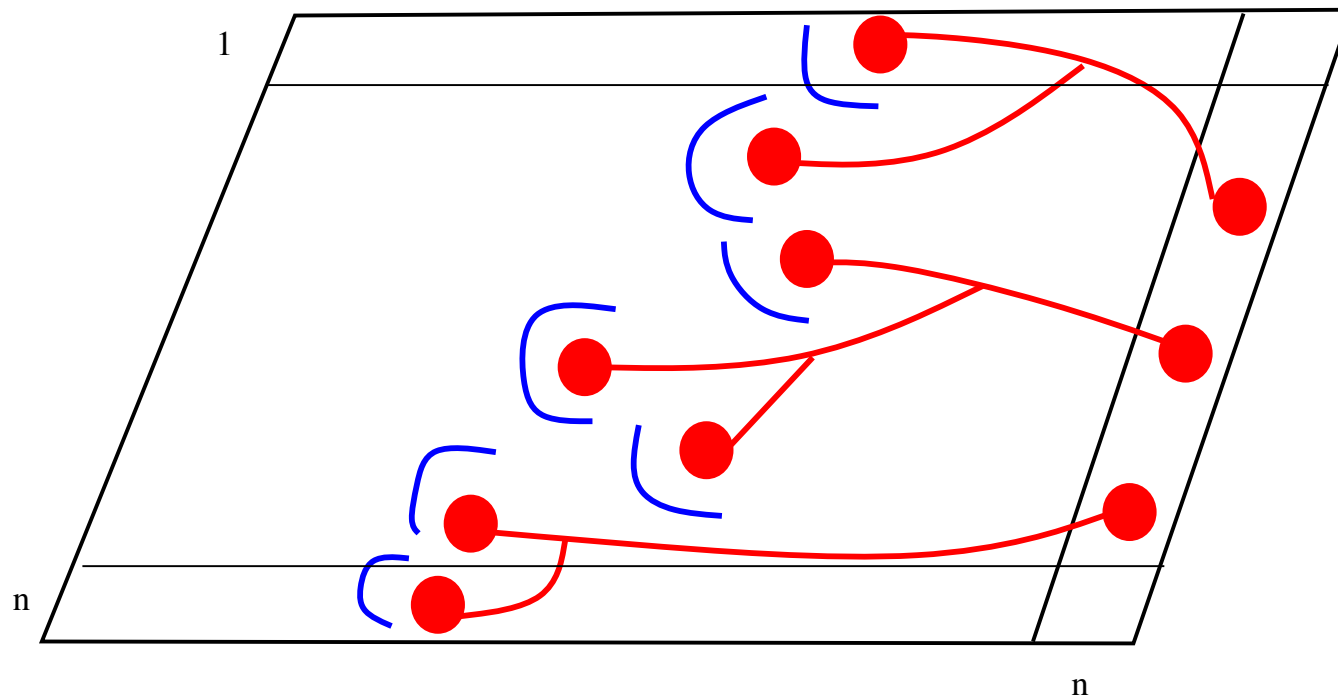


Illustration of the ideas (2/3)

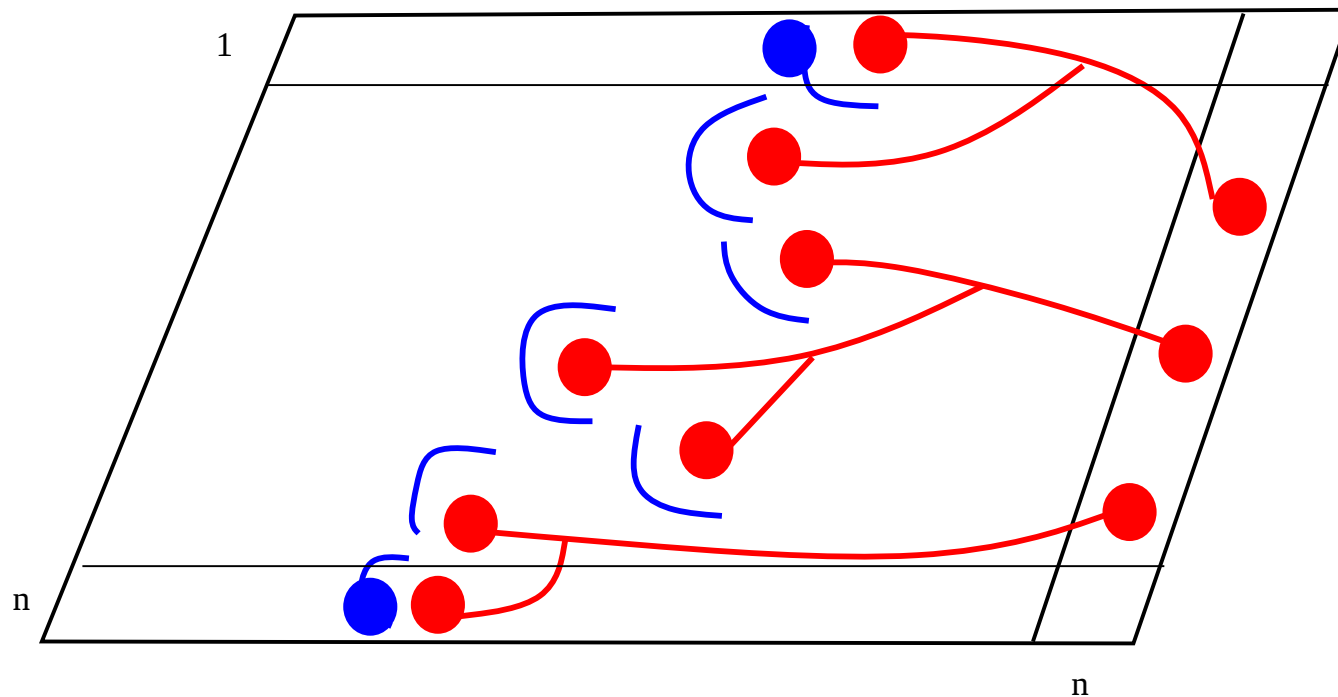
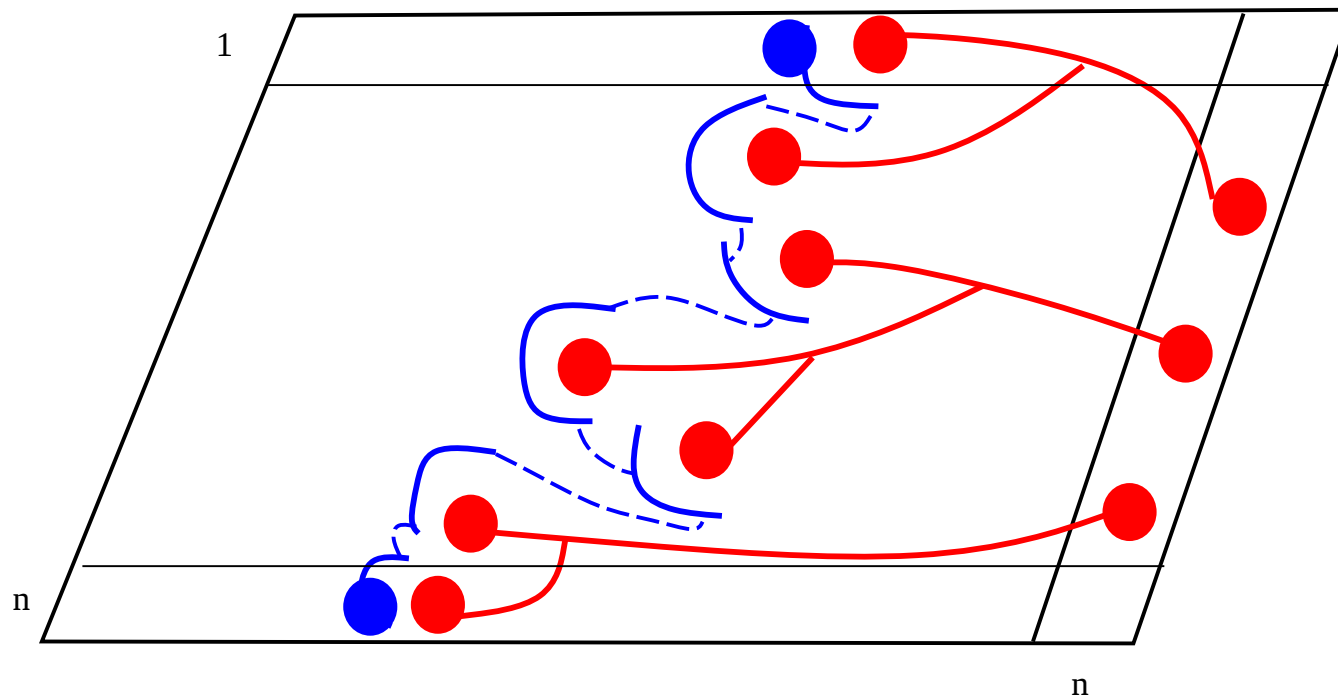


Illustration of the ideas (3/3)



Property 3: Hex is first-player-win

- Proof is constructed using the “**strategy-stealing**” argument made by John Nash in 1949.
 - If there is a winning strategy for the second player, the first player can still win by making an arbitrary first move and using the second-player strategy from then on.
 - ▷ *The first player ignores the arbitrary first move by assuming that move does not exist.*
 - ▷ *Hence the second move made by the second player becomes the first move.*
 - ▷ *The third move made by the first player becomes the second move.*
 - If using the second-player strategy requires playing the chosen first move or any move played before, then make another arbitrary move.
 - ▷ *An arbitrary extra move can never be a disadvantage in Hex.*
 - We have obtained a contradiction, and thus the second player cannot win from the initial empty board.
 - Since we have proved there is no draw, and there is always a winner, and both players cannot win at the same time, the first player must have a winning strategy from the initial empty board.

Strategy-stealing argument: proof (1/3)

■ Definitions

- The first player is R and the second player is B .
- The initial board position is P_0 which is an empty board.

■ Assume the second player B has a winning function $f(P)$ that tells the next ply towards winning when seeing the position P .

■ Problems for R to “use” $f(P)$.

- $f(P)$ has a value only for the case P is a legal position for the second player only.
 - ▷ P is constructed from P_0 assuming R can place any ply he wants.
- $f(P)$ returns the x - y coordinates of a location and the color of the piece to play which is B .

Strategy-stealing argument: proof (2/3)

■ Definition

- $rev(m)$: flip the color and coordinate of a ply m .
 - ▷ Let $m = (x_m, y_m)$ be the location to play.
 - ▷ Let c be the color of the piece to play.
 - ▷ Let $\bar{c}$ be the color flipped.
 - ▷ Return the location (y_m, x_m) and the color $\bar{c}$ where $\bar{B} = R$ and $\bar{R} = B$.

■ The steps taken by the first player R to win using $f(P)$:

- R makes an arbitrary 1st ply m_1 . Store this move into the variable m^* .
- B makes the 2nd ply $m_2 = f(P_0 + m_1)$.
- R makes the 3rd ply $m_3 = rev(f(P_0 + rev(m_2)))$.
 - ▷ If $m_3 = m^*$, then make another arbitrary ply and store it to be the new value of m^* .
- B makes the 4th ply $m_4 = f(P_0 + m_1 + m_2 + m_3)$.
- R makes the 5th ply $m_5 = rev(f(P_0 + rev(m_2) + rev(m_3) + rev(m_4)))$.
 - ▷ If $m_5 = m^*$, then make another arbitrary ply and store it to be the new value of m^* .
- B makes the 6th ply $m_6 = f(P_0 + m_1 + m_2 + m_3 + m_4 + m_5)$.
- ...

Strategy-stealing argument: proof (3/3)

- Hence we know the first player also has a winning connection.
- We also know the followings.
 - The game is finite and deterministic.
 - There is at least one winner before or when the board is completely filled.
 - There cannot be two winners.
 - Hence we can enumerate the whole solution search tree.
 - ▷ *In this solution search tree, there is a way for one player to win all of the times no matter what the opponent reacts.*
- Thus the assumption of the second player having a winning strategy f is not valid.

Strategy-stealing argument: comments

- This is not a **constructive** proof.
- It only shows the first player has a winning strategy from the initial empty board, not from an arbitrary position.
- The strategy-stealing argument may not be good for other games.
- The argument works for any game when
 - there is a way for the first player not to lose at the first ply,
 - the second player cannot win at the second ply,
 - it is symmetric,
 - it is **history independent**,
 - it always has exactly one winner, and
 - ▷ *namely, it cannot have a draw by having no winner or two winners,*
 - **an arbitrary extra move can always be made and can never be a disadvantage.**
 - ▷ *Note: it requires that a player is always possible to place an arbitrary move which may not be true for some games.*

Properties of Hex

■ Variations of Hex

- The **one-move-equalization** rule: one player plays an opening move and the other player then has to decide which color to play for the remainder of the game.

▷ *The revised version is a second-player-win game (ultra-weakly).*

■ Hex exhibits considerable mathematical structure.

- Hex in its general form has been proved to be PSPACE-complete by Even and Tarjan in 1976 by converting it to a Shannon switching game.
- The state-space and decision complexities are comparable to those of Go on an **equally-sized** board.

■ Solutions

- (Weakly or strongly) solved on a $6 * 6$ board in 1994.
- Maybe possible to solve the $7 * 7$ case.
 - ▷ *The $7 * 7$ case was solved in 2001. [Yang et al. 2001]*
- Not likely to solve the $8 * 8$ case without fundamental breakthroughs.
 - ▷ *The $8 * 8$ case was solved in 2009. [Henderson et al. 2009]*
- The $9 * 9$ case was solved in 2013. [Pawlewicz et al. 2013]
- The most-central opening was solved for the $10 * 10$ case in 2014. [Pawlewicz et al. 2013]

More divergent games (1/3)

- **Polyomino games:** placing 2-D pieces of a connected subset of a square grid to construct a special form.
 - Pentominoes
 - Domineering
 - Games on smaller boards have been solved.
- **Othello**
 - M. Buro's LOGISTELLO beat the resigning World Champion by 6-0 in 1997.
 - Weakly solved on a $6 * 6$ board by J. Feinstein in 1993.
 - ▷ *Second-player-win*
 - Weakly solved on a $8 * 8$ board by Hiroki Takizawa in 2023.
 - ▷ *Draw: a fair game*
- **Chess**
 - DEEP BLUE beat the human World Champion in 1997!

More divergent games (2/3)

■ Chinese chess

- Professional 7-dan since 2007.
- Still in progress, but lack of reporting recent advancement due to complications facing the using of deep learning based approaches.

■ Shogi

- Claimed to be professional 2-dan in 2007.
- Defeat a lady professional player in 2010.
- Defeat a 68-year old 1993 Meijin during 2011 and 2012.
- Still in progress, but lack of reporting recent advancement due to complications facing the using of deep learning based approaches.

More divergent games (3/3)

■ Go

- 5 by 5 Go was weakly solved in 2003 [Werf et al '03] and strongly solved in 2024 [Hsu et al '24].
 - ▷ *First player wins and takes all cells using 22 plys.*
- Recent success and breakthrough using Monte Carlo UCT based methods between 2004 and 2012.
- Amateur 1 – 4 kyu in 2008.
 - ▷ *Beat a professional 8-dan by having an 8-stone advantage.*
 - ▷ *Beaten by a professional 9-dan by giving a 7-stone advantage.*
- Amateur 1 dan in 2010.
- Amateur 3 dan in 2011.
- The program Zen beat a 9-dan professional master at March 17, 2012.
 - ▷ *First game: Five stone handicap and won by 11 points.*
 - ▷ *Second game: four stones handicap and won by 20 points.*
- Go (19 by 19) is now over human: AlphaGo beat a human top player by a margin of 4:1 at March 2016, and beat the human top player by 3:0 at May 2017.

Table of complexities

Game	$\log_{10}(\text{state-space})$	$\log_{10}(\text{game-tree size})$
Nine Men's Morris	10	50
Pentominoes	12	18
Awari	12	32
Kalak(6,4)	13	18
Connect-four	14	21
EinStein würfelt nicht!	15	22
Domineering (8 * 8)	15	27
Dakon-6	15	33
Checkers	21	31
Othello	28	58
Qubic	30	34
Draughts	30	54
Chinese Dark Chess	37	135
Chess	46	123
Chinese chess	48	150
Hex (11 * 11)	57	98
Shogi	71	226
Renju (15 * 15)	105	70
Go-Moku (15 * 15)	105	70
Connect6 (19 * 19)	172	70
Go (19 * 19)	172	360

State-space versus game-tree size

- In 1994, the boundary of solvability by complete enumeration was set at 10^{11} .
 - The current estimation is about 10^{13} (since the year 2007).
- It is often possible to use heuristics in searching a game tree to cut the number of nodes visited tremendously when the structure of the game is well studied.
 - Example: Connect-Four.
 - Good heuristics for some games are easier to design than the others.

Remarks

- **With the same complexities, a stochastic game is usually more interesting and challenging than a deterministic game.**
 - Example 1: EinStein würfelt nicht! (EWN) is definitely more challenging than Connect Four.
 - Example 2: Chinese Dark Chess (CDC) is more interesting than Chess.
- **Rules of playing games also affect the fundamental properties of games.**
 - Example: in Shogi you own the captured pieces and may turn them into yours in later plays.

Methods developed for solving games

■ Brute-force methods

- Retrograde analysis
- Enhanced transposition-table methods

■ Knowledge-based methods

- Threat-space search and λ -search
- Proof-number search
- Depth-first proof-number search
- Pattern search

- ▷ To search for *threat patterns*, which are collections of cells in a position.
- ▷ A threat pattern can be thought of as representing the *relevant area* on the board, an area that human players commonly identify when analyzing a position.

■ Recent advancements:

- Monte Carlo UCT based game tree simulation.
 - ▷ Monte Carlo method has a root from statistic.
 - ▷ Biased sampling.
 - ▷ Using methods from machine learning.
 - ▷ Combine domain knowledge with statistics.
- Combine searching with deep learning.

Brute-force versus knowledge-based methods

- Games with both a relative low state-space complexity and a low game-tree complexity have been solved by both methods.
 - **Category 1**
 - Connect-four and Qubic
- Games with a relative low state-space complexity have mainly been solved with brute-force methods.
 - **Category 2**
 - Namely by constructing endgame databases
 - Nine Men's Morris
- Games with a relative low game-tree-complexities have mainly been solved with knowledge-based methods.
 - **Category 3**
 - Namely, by intelligent (heuristic) searching
 - Sometimes, with the helps of endgame databases
 - Go-Moku, Renju, and k -in-a-row games
- Games in Category 4 are difficult to solve and may be solvable using learning based methods.

Advantage of the initiative

- Theorem (or argument) made by Singmaster in 1981: The first player has advantages.
 - Two kinds of positions
 - ▷ *P-positions: the previous player can force a win.*
 - ▷ *N-positions: the next player can force a win.*
 - Arguments
 - ▷ *For the first player to have a forced win, just one of the moves must lead to a P-position.*
 - ▷ *For the second player to have a forced win, all of the moves must lead to N-positions.*
 - ▷ *It is easier to the first player to have a forced win **assuming all positions are randomly distributed**.*
 - ▷ *Can be easily extended to games with draws.*
- Remarks:
 - On small boards, the second player is able to draw or even to win for certain games.
 - Cannot be applied to an infinite board.
 - The assumption of all positions are randomly distributed may not be true for all games.

How to make use of the initiative

- A potential universal strategy for winning a game:
 - Try to obtain a small advantage by using the initiative.
 - ▷ *The opponent must react adequately on the moves played by the other player.*
 - To reinforce the initiative the player searches for threats, and even a sequence of threats using an evaluation function E .
 - Force the opponent to always play the moves you expected.
- Threat-space search
 - Search for threats only!

Offsetting the initiative

- An example of a game with a huge initiative:
 - A connection $mn1$ -game.
 - ▷ *Connect-1 was mentioned in a famous Sci-fi novel published 1979.*
 - A connection $mn2$ -game.
 - A connection $mn3$ -game.
 - For a connection mni -game, you can have a feeling that the advantage given to the first player through initiative is gradually lessened when i gets larger.
- Need to offset the initiative.
 - The offsetting rule must be simple.
 - The revised game must be as **fair** as possible.
 - ▷ *It is difficult to prove a game is fair.*
 - ▷ *Example: Paper-scissor-stone is fair.*
 - The revised game needs to be fun to play with.
 - The revised game cannot be too much different from the original game.
- Knowing how to properly offsetting the initiative may uncover some fundamental properties of the game such as its **level of difficulty**.

Examples (1/2)

- Enforce rules so that the first player cannot win by selective patterns.
 - Renju.
 - ▷ *Still first-player-win.*
 - Go ($19 * 19$).
 - ▷ *The first player must win by more than 7 stones.*
 - ▷ *Komi = 7.5 in 2011.*
 - ▷ *The value of Komi changes with the time and may be different because of using different set of rules.*
- The **one-move-equalization** rule: one player plays an opening move and the other player then has to decide which color to play for the remainder of the game.
 - ▷ *Hex*
 - ▷ *Second-player-win if Hex uses this variant*

Examples (2/2)

- The first move plays one stone, the rest plays two stones each.
 - ▷ *Connect6.*
 - ▷ *Intuitively, in each turn the initiative goes to different players alternatively.*
 - ▷ *Still not able to prove the game is fair (upto the year 2021).*
- The first player uses less resources.
 - For example: using less time.
 - ▷ *Chinese chess.*
 - A resource-auctioning scheme.
- General rules to redesign a game to make it fair are unknown which makes a very good and challenging research topic.

Conclusions

- The knowledge-based methods mostly inform us on the structure of the game, while exhaustive enumeration rarely does.
- Many ad-hoc recipes are produced currently.
 - The database can be used as a corrector or verifier of strategies formulated by human experts.
- Monte-Carlo searching technique with deep learning seems to open a new avenue of researching.
- It may be hopeful to use data mining techniques to obtain cross-game methods.
 - Currently not very successful.
 - It is noted that Alpha Zero claimed on having a universal solution by using techniques from deep learning.

Comments

- Can combine knowledge-based method with exhaustive enumeration.
 - For converging games, build endgame databases when the remaining state spaces is manageable.
 - ▷ *Example: build endgames with at most 5 pieces in Chess and stop searching when the number of pieces on the board is less than 6.*
 - For diverging games, pre-compute all possible opening moves and solve them one by one in sequence or in parallel.
- This is different from the usage of pattern databases in solving one-player games.
 - Patterns are used to guide the search in solving one-player games.
 - Endgame databases are used here to stop the search earlier which has a flavor like that of bi-directional search.

1990's Predictions — 2000's Status

- Predictions were made in 1990 [Allis et al. 1991] for the year 2000 concerning the expected playing strength of computer programs.

solved	over champion	world champion	grand master	amateur
Connect-four	Checkers (8 * 8)	Chess	Go (9 * 9)	Go (19 * 19)
Qubic	Renju	Draughts (10 * 10)	Chinese chess	
Nine Men's Morris	Othello		Bridge	
Go-Moku	Scrabble			
Awari	Backgammon			

- Very successful in using exhaustive enumeration.
- color code
 - Green: Performs much better than expected
 - Red: right on the target.
 - Black: have some progress towards the target.
 - Blue: not so.

Predictions for 2010

- Predictions were made at the year 2000 for the year 2010 concerning the expected playing strength of computer programs.

solved	over champion	world champion	grand master	amateur
Awari	Chess	Go (9 * 9)	Bridge	Go (19 * 19)
Othello	Draughts (10 * 10)	Chinese chess	Shogi	
Checkers (8 * 8)	Scrabble	Hex		
	Backgammon	Amazons		
	Lines of Action			

Predictions for 2010 – Status

- My personal opinion about the status of Prediction-2010 at October, 2010, right after the Computer Olympiad held in Kanazawa, Japan.

solved	over champion	world champion	grand master	amateur
Awari	Chess	Go (9 * 9)	Bridge	Go (19 * 19)
Othello	Draughts (10 * 10)	Chinese chess	Shogi	
Checkers (8 * 8)	Scrabble	Hex		
	Backgammon	Amazons		
	Lines of Action			

- Successful in using knowledge based methods.
- color code
 - **Red**: right on the target.
 - Black: have some progress towards the target.
 - **Blue**: not so.

Concluding remarks

- **Very diverse development since year 1940 until 2010. New methods and techniques are discovered every couple of years.**
 - Accompany the development of Computing Science.
- **Between years 2010 and 2020, a lot of progress was achieved in Go-related research.**
 - Mostly using deep learning techniques.
 - Mostly on better winning games.
 - Not much is done with regard to pure game-theoretical researches.
- **A better link of game related research and bettering human life is needed.**

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